ANALOGS YESTERDAY, TODAY, AND TOMORROW

OR: METAPHORS OF THE CONTINUUM

by

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ORIGINS

It was naturally pleasurable for me to be approached by the Simulation Councillors to write an article for the new magazine. This euphoria persists even now, when my performance has in fact begun, and is only moderately tempered by the haunting suspicion of what their real reason might have been for so honoring me. It certainly could not be because my views on analog computing and simulation are somewhat eccentric in relation to much contemporary doctrine, although I accept — and actually relish — this characterization. It could conceivably be in recognition of my relatively early start in the field of electronic analog technology; this again is not denied by me, but here we may have found the clue. The fact that I began a long time ago in this sort of activity doesn't mean at all that I am either oracle or authority. The truth of the matter is subtler still: it only means that I am getting old. So we have it out at last. They are showing respect for the aged. Here then, steeped in mellow nostalgia, are the musings of a well-meaning and harmless Old Timer.

Since truth will out, I might as well admit that I do not claim to be the original inventor of the operational amplifier.* It is true, however, that I did build some of them more than four years before hearing of anyone else's, and that their purpose was truly simulative. These amplifiers were indeed DC feedback units, used to perform mathematical operations in an analog structure, but the very first such amplifier *itself* began as a model of a mechanical control amplifier. Thus my role as model builder, even at that stage, loomed larger than my possible role as inventor, and I have been dealing continually with models and analogs ever since.

I shall make no pretense in this article of assigning credit to other individuals or institutions. There are far too many of both involved to give any accurate and fair account of the brainpower and perspiration which have made analog computing what it is today.

While electronic analog equipment certainly existed in the thirties — and in the forties became available on the open market — its roots really go further back in time. It is doubted that an exhaustive chronology of the contributory precursor technologies could ever be produced, let alone by one amateur historian, but it is hoped that an outline of the tools and techniques on hand in the previous era will show that the ingredients were already there and that the modern analog machine was almost inevitable.

*Shall we define operational amplifier as one specifically designed to carry out mathematical operations? —Ed.

Several fields of science and engineering overlapped to breathe life into this development: Physics and Scientific Instruments, Communications and Electronics, Controls and Servomechanisms, Mathematics, and Aeronautical plus Electrical plus Mechanical Engineering. There is one thread, come to think of it, which appears to run through the whole background of the analog doctrine, and which may be said to belong to it more intrinsically than it does to the other major branch of computation: that thread is *feedback*. It will appear again frequently in what follows.

The clearest anticipation of modern analog machines was in the differential analyzer. This primarily mechanical device could handle total differential equations at least as well as we can now, and in some ways better. One such analyzer afforded automatic establishment of its interconnections and parameters, tape storage of these data, and automatic readout, both numerical and graphical. Although slower than newer electronic equivalents, nonetheless for a 19-integrator problem run on it in 1945 — a thoroughly nonlinear problem, by the way — the analyzer time scale was only twice as slow as the real scale for the remotely controlled glide vehicle being simulated. The disc integrators of this machine were things of beauty, with accuracies approaching, and resolution exceeding, five decimals. They could integrate with respect to dependent variables, thus enabling multiplication with only two integrators, logarithms without approximation, and so on. Integrators of this same general type were also applied in astronomical and military computing devices.

This type of equipment inspired many of the electronic analog devices which followed, as well as the digital differential analyzers. Although the electronic integrators of the analog equipment prefer time as the direct variable of integration, they have shown extreme flexibility of operating speed. Imagine the mechanical discs of the older analyzers running at millions of rpm trying to keep up with their progeny!

The disc integrators of the differential analyzer worked without feedback, as did its other basic parts. Where then did feedback appear in these analyzers? In the differential equations acted out within them. Any equation requiring solution involves at least one causal loop. But for feedback in its more exuberant forms we nominate automatic controls.

Regulatory mechanisms such as those found in industrial control systems have been around for a long time. In roughly historical sequence, they have been mechanical, hydraulic, pneumatic, electric, and electronic. Translating as they do from the unbalance, or error, in a controlled condition, to the manipulation which is intended to reduce the unbalance, they close a feedback loop which includes some kind of plant. In typical cases these mechanisms have embodied mathematical laws with continuous fidelity, to attain which they have resorted to internal feedbacks precisely analogous to those employed in a modern operational amplifier. This sort of local feedback was applied in standard controller mechanisms of the twenties and even earlier. As these antecedent regulatory devices qualify as DC feedback amplifiers in every sense, with feedback and even null-seeking at two distinct levels, and with mathematical capabilities, it is not difficult to trace the logical paths of evolution from these devices to analog computing as it is now enjoyed.

It was not uncommon in the thirties to build simulators embodying convenient models of plants, into which the real regulatory mechanisms could be connected. Both developmental and educational purposes were served by these structures, just as with simulators today.

The next stage, in which the real control mechanisms were replaced by models, permitted the whole loop to be electronic and hence vastly more flexible. In such simulators several plants might be interconnected so that the newer stability problems thus encountered could be studied conveniently. Again plants with multiple inputs and outputs having internally interacting paths were included, and regulatory loops in hierarchies where master controls manipulated the desired conditions of subordinate controls were simulated. Note the ascending succession of feedback loops: within amplifiers to attain promptness and stability; locally around amplifiers to give the desired mathematical performance for regulatory mechanisms; in control loops to promote the minimum difference between desired and existing conditions; in more comprehensive control loops which include complete but subordinate loops in cascade; in still more comprehensive loops for supervisory or evaluative purposes; and finally in experimental design and optimizing operations, using models or computational structures to evolve more effective system operation.

Servomechanisms are also part of the lore which preceded and inspired the modern analog machines. Though not as old as the governors, regulators, and controllers of temperature, flow, level, etc. previously mentioned, servos as positional followers were functionally similar in control philosophy and feedback loops. Furthermore, being more modern, they benefited from the increasingly mathematical technologies of design and development. Perhaps most relevant was the simultaneity and parallelism between servo theory and that of feedback amplifiers in communications. Stability criteria for the latter were seen as applicable to the former, at least in the linear realm. Analysis in the frequency domain, a natural procedure for linear communications equipment, was carried over rather directly to servomechanisms. This debt has since been partially repaid, as servomechanisms have helped to furnish nonlinear analog elements and other computing equipment for the study of nonlinear phenomena, generally in the time domain.

Thus do the various doctrines and practical disciplines feed on each other to mutual benefit, and (if you will forgive the liberty) feed sideways as well as back and forth.

We pick up servomechanisms again, much farther back along the trail. Though scientific instruments do practically everything today, including computation, synthesis, manipulation, and regulation, they were once used principally for measurement. For accurate measurement it was found that feedback methods, when possible, were surpassingly effective. While the underlying philosophical reasons for this circumstance are of vital importance, we shall take them here on faith. Note, however, that the observation of balance in a measurement, and the manipulation which may be made to achieve it, is still a feedback process even if done by a human agency; the slave can be the experimenter himself. Precise weighing with a beam balance may stand as a clear example of this procedure, but a myriad of others may readily be spread forth.

Succinctly, the process is reduced by feedback to dependency on only one, or a few, reliable elements. Automation of the loop-closing, null-seeking action merely replaces one slave by another. In this light the venerable self-balancing slidewire potentiometer recorder stands with the latest feedback operational amplifier.

Antedating, but partly anticipating, the development of active analogs was the use of models which depended much more directly on the analogies between phenomena as they appear in widely differing physical media. Of main concern here are those cases in which the modeling medium has been electric, but quite accurate and articulate models have also been mechanical and hydraulic, and many of these are hoary with age. Ever since accurate and dependable circuit elements have been available (and this has been for many decades, notably for resistors and capacitors), highly successful passive models have been built for the study and solution of such problems as those which occur in heat conduction. Dynamic as well as steady-state phenomena may be handled, often in the same model. Again, vibrations have been studied with direct models having all three kinds of circuit element, plus transformers. Furthermore very large and complete simulative structures, called network analyzers and based heavily on passive elements, have been used, in particular for AC power distribution and communication lines. Even today one finds such continuous conductive models as electrolytic tanks still in use and under development.

Many of these tools have specialized capabilities hard to match with more modern apparatus. The very demanding doctrines of "lumping," which must take place when continuous systems are represented by separate but connected analog operations, are substantially unchanged as compared to those in passive models. Here is another branch of knowledge and effort, then, to which we owe recognition as contributing to present-day simulation and computing.

From a different direction came another practical model-building technique. This one is straight down the simulation highway: we refer to trainers like those used for many years to indoctrinate pilots of aircraft. These trainers modeled just about everything except nonangular spatial accelerations. They presented to the human operator a simulated environment resembling the real one in many important ways, including his manipulations and the responses returned to him as a consequence thereof. Of course the later counterparts of the first training aids have become more refined, and similar structures have been adapted to other man-machine collaborations, * but the inspiration to analog enthusiasts on a broader scale seems rather obvious. Here was an operative model, in real time and undelayed, where to the sensory and motor periphery of the trainee the real environment was presented in a safe and pedagogically corrective atmosphere.

Now it is true that training devices for physical skills are even more numerous today, and analog simulative equipment finds important applications in these, but a somewhat extended simile might be in order. For in system design in its larger implications we are all trainees; analog simulation, to teach us how a proposed system might work when at least part of it is new, to guarantee safety if we try out a poor idea, and to offer peripheral communication at the deliberative level, projects the trainer concept to an advanced modern setting. The task of simulating the pilot or other human operators provides a challenge which has only partly been met, and which is still relevant. Simulating the system designer, as a logical extension, leads as far as you might care to travel.

OVERLOOK

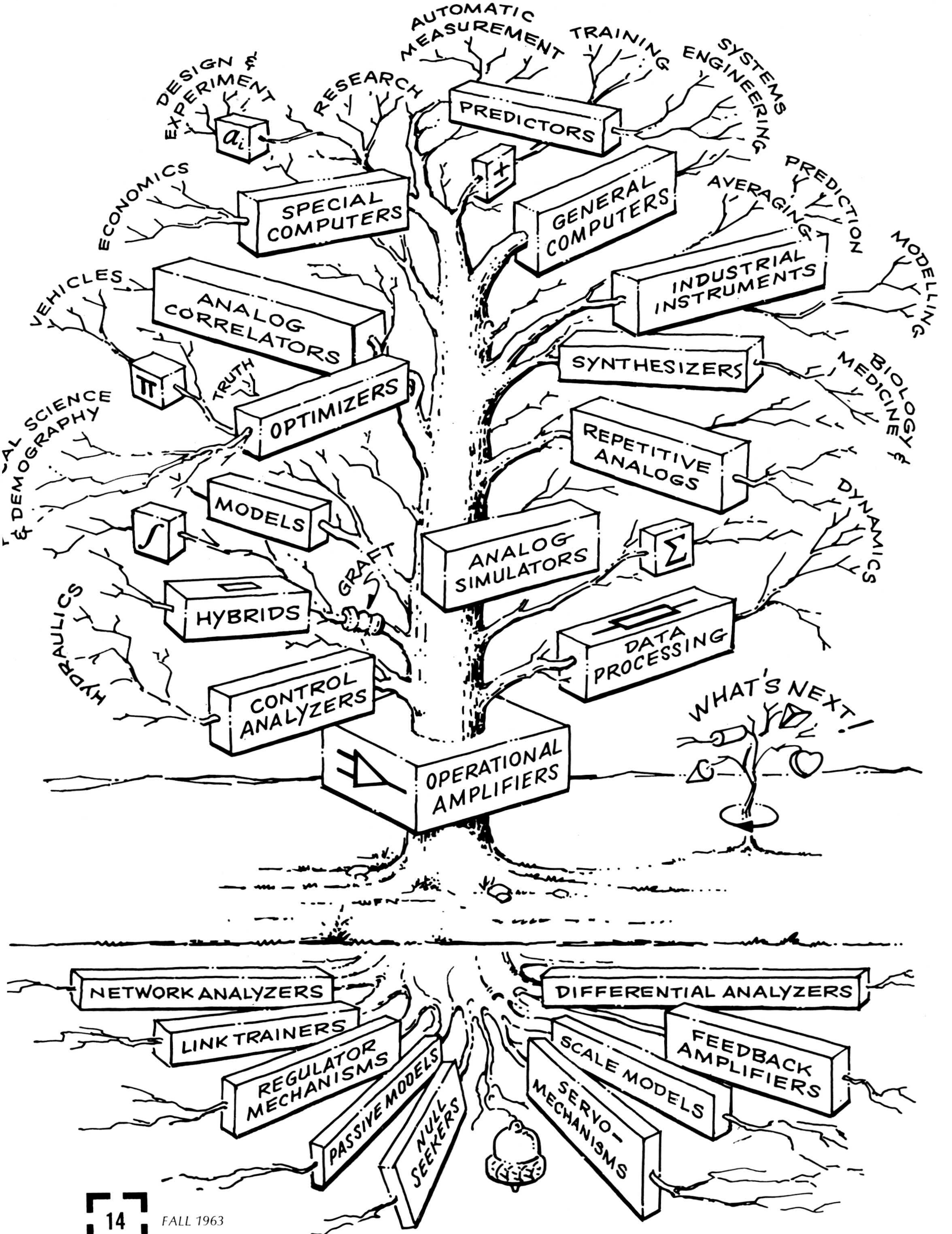
Things are looking up everywhere for the analog profession. Substantially every branch of engineering now applies analog computing equipment: theory, experiment, design, manufacture, and test. Applications are even on the increase for scientific research. We shall not try to list the many embodiments and applications in this text, but have included some of them in the accompanying figure, prepared to bear out the morphology of our burgeoning field.

Analog representation in terms of modern apparatus is a far cry from scale models, but the model concepts still seem incomparably fruitful. In direct models, which retain the physical medium of their prototypes, scaling is the most important factor. Similitude conditions must be faithfully adhered to, and an appreciation of these conditions imparts a feeling for models which is never lost. Actually the use of direct scale models has not decreased, and is still a powerful technique in such areas as hydraulics and structures, natural and man-made. Much ingenuity has been lavished on such models; they must by no means be looked down upon by the designers and users of more fashionable modeling items.

In a scale model the transformation of dimensions is typically direct and simple, especially if shape is preserved. Even when the scaling involves distortions of shape, such as relative compression and bending, the transformations generally carry distance into distance, velocity into velocity, and so on, with only numerical scale factors relating them in pairs. Basic parameters, when the scale ratios are properly assigned, turn out also to be numerical, and apply equally to model and to prototype. This doctrine, whereby characteristic system parameters are dimensionless, is applicable to all modeling procedures.

The transformation concept, so clear and concise for scale models, carries over with little confusion to modeling in which the physical form is changed, and ultimately to electronic analogs where transformation includes transmogrification. The scale ratios in general, however, are no longer numbers, but the basic parameters may be. This kind of introduction is recommended for physicists and applied mathematicians who may be coming suddenly into modern analog contacts, since it utilizes some of the ideas and precepts of the more classical fields. ⟶

* See "Manned Spacecraft Simulation" in this issue of *SIMULATION* —Ed.


AUTOMATIC MEASUREMENT
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DATA PROCESSING
CONTROL ANALYZERS
WHAT'S NEXT!
OPERATIONAL AMPLIFIERS
NETWORK ANALYZERS
DIFFERENTIAL ANALYZERS
LINK TRAINERS
FEEDBACK AMPLIFIERS
REGULATOR MECHANISMS
SCALE MODELS
PASSIVE MODELS
NULL SEEKERS
SERVO-MECHANISMS

Another sort who is momentarily taken aback at the liberties permitted in analog models is an engineer who has been too long away from the time domain. Often brought up on linear systems and frequency analysis, he may be suspicious of a mechanism which gives solutions as functions of time, perhaps not realizing that it will provide amplitude and phase spectra as well, if one merely applies a different stimulus to the same model structure. It is frequently worth while, in these cases, to introduce the analog from the viewpoint of the frequency domain, shifting later to the strange and magical.

Oddly enough, the most confirmedly practical and the most profoundly theoretical of engineers will both be found to favor the time domain, with or without computing equipment. In the former case this is by virtue of convenience in handling real equipment; in the latter because — among other reasons — he finds it better to approach nonlinear problems in the time rather than in the frequency domain.

Analog engines have not always been as respectable as now. Analogy itself we have been warned against, in proverb and in folklore, as being dangerous and requiring proof. Simulation has had connotations of deceit, empiricism, and quackery. It was stylish, even recently, to say that the only good electronics is that which says Yes or No. There is nothing to be gained in disputing these allegations, least of all by excited rejoinder. The continuous active analog is in its infancy, and time is (literally) running in its favor!

Time as an independent variable, furnished at low cost by Nature, has the advantage of nearly (if not actually) infinite resolution. This continuity, coupled with the continuity of voltage and charge, leads to the ability to close loops at very high frequency, or with short time intervals. As a consequence one may approach the ideals of differentiability which are inherent in the infinitesimal calculus, which postulates the existence of a continuum. While most contemporary analog apparatus does not press these limits, it is comforting to know that there is room left in which to maneuver.

In modest applications to on-line measurement and data processing, it is quite generally conceded that the advantages of continuous analog apparatus make it irresistible. This is owing partly to the simplicity and speed which its continuity makes possible, and partly to the fact that almost every input transducer is also "analog" in character, that is to say, continuous in excursion and time. Storage and sampling, for example, are frequently unnecessary.

When we turn from simpler to more involved data processing, to ambitious simulation, or to more pretentious computations, there has been some feeling that digital means should automatically be substituted for analog, especially if funds are available. In this connection we should like to quote (admittedly out of context) no less a figure than Dr. Simon Ramo:

*"Digital computers, however, cannot be used conveniently or efficiently to obtain answers to all of the problems. In some cases, even they cannot solve the equations in any reasonable time, and in other cases the problems are not understood well enough for satisfactory mathematical formulation. Under these circumstances we can often turn to analog, real-time, simulation devices to predict the behavior of the system. No engineering computing center is well equipped without such devices."**

One should certainly be happy to settle for this, even though the text continues with a discussion of other kinds of equipment than analog with which the latter may be associated. Only the most hard-shelled of analog champions would suggest that all simulative and computational equipment be undiluted by numerical or logical adjuncts. Certainly many of the best known individuals and organizations in the analog field are now willing and able to talk about hybrids.

At a large Eastern university, under the guidance of a well-known and gifted computationalist, a successful project has been reported in which the scaling for an analog installation is done entirely by rote on a digital machine. No guessing or trial runs are involved. Straight from the equations, the digital solution dictates the analog settings which will bring the maximum excursion of every variable analog voltage to within 20% of the limiting value. Local wags thus proclaim the discovery at last of a practical contribution by the digital apparatus. Seriously, what they like about this use of digital machines is that it preserves the ability to "get at" the analog solutions during operation.

While it is agreed that analog and digital techniques will increasingly cross-fertilize and interrelate, it is predicted that the controversy between their camps will rage on, goodnatured but unbated, for years to come. The serious issue of reliability has recently arisen between the two ideologies, referring — for example — to instruments for interplanetary exploration. It is preferred here to avoid an opinion or judgment on this very important issue, and it is suggested that others similarly withhold judgment. At all costs we must not go down the wrong road; there are quite powerful and rational and experienced

*Writing on Systems Engineering in *Parts and Wholes*, Daniel Lerner, Ed., Macmillan, New York, 1963.

brains in which the reliability vote would be cast for analog, or at least against the exclusion of continuous variability. We must cooperate in a dispassionate but devoted study to determine the likeliest facts and fancies in this affair. If one believes that Nature is ahead in reliability, and there would appear to be justification for this belief in recognition of the redundancy, repairability, and adaptability of animal organisms, then conclusions may follow which are based on how one views such organisms. It has been standard practice to view the details of animal nervous systems as evidence that they are digital, but there are major reasons to question this.* The central nervous system itself seems digital to digital men, and analog to analog men. If it is both, then it is a more intimately and profoundly intermingled hybrid than any of the artificial structures which have come to light. One thing is sure, and that is that the brain builds models. We are in good company!

AUGURIES

The analog-digital struggle will persist, and this will mean some wear and tear as the proponents contend, but such contention will probably be beneficial since it should assure that the maximum potential of each technique will be realized. As to mixtures, all the obvious ones will soon be seen somewhere. More intimate mixtures, which might offer something approaching universal applicability, will depend on the appearance of new instrumental tools. But urgent needs provide as potent a force for development as does the availability of new and startling techniques. Hasty prediction from either angle would be hazardous; certainly anything specific on our part would be irresponsible as well as foolhardy.

There do seem to be possibilities, however, in recognition of the ability of continuous analog instruments to operate quickly and smoothly in closing feedback loops, plus the arbitrary accuracy and permanence of discrete processes. Graphical computation may give a clue here, since anyone who deals with geometrical plots is prone to appeal alternately to continuous criteria and to numerical coincidences in calibration.

As to analogs themselves, it is evident that some forms of instruments will become progressively smaller and handier in solid-state incarnations. It is also evident that optimizing and search operations will be made increasingly automatic, as the deliberative functions of the user are encroached on more and more by deliberately imposed autonomous controls. But one of the principal lessons from the past is that substantially all of the earlier techniques will continue to be used, and will grow and improve horizontally. You possibly have a slide rule in your pocket, though admittedly you may have turned in your abacus for a desk calculator.

It would be a big surprise if passive electric models do not expand in application and in technical excellence. More adept peripheral instruments, to drive and to measure them, are either in the cards or on the table. Passive circuit elements, adjustable as well as fixed, are improving as to accuracy, bandwidth, and stability. In this category are included not only resistors, capacitors, inductors, and transformers, but also certain nonlinear elements. A combination of compensation and regulation can cut the parametric effects of temperature down to size, especially with the advent of flexible devices for thermoelectric heat pumping. Relatively little work has been done on passive networks for model building compared to that expended for communications, even for linear systems. The challenges introduced by the nonlinear cases are considerable, but with newer analytical techniques and instrumental tools it would be unwise to put limits on what might be accomplished. Part of the lure is that many biological structures appear to have been designed along these lines.

Another trend which is evident and will probably gain in momentum is the unification of assorted instrumental techniques based on analog feedback operations. When it is considered how fundamental is the function of the operational amplifier, and how its benefits are continually being rediscovered in new fields of technology, it seems likely that multipurpose modular structures will perform the tasks of a number of specialized measuring and manipulative instruments. Beyond the classical and celebrated mathematical operations of the amplifier (comprising addition, algebraic and functional inversion, linear combination, differentiation, integration, etc.) are the abilities to store and to isolate, among others less well known.

The philosophy of this type of amplifier as an electrical null-seeking or balancing agent carries its own impact once it is understood. When basically similar methods and equipment are found to be effective in each, such fields as computing, data processing, testing, regulation, and model building will not be kept separate but will diffuse and perhaps ultimately fuse with one another. One key to the future appears to lie in the quasi-paradox of special-purpose instrumental assemblages based on general-purpose analog modules.

Systems engineers are coming along now in greater numbers and of higher average caliber — and they are not now so brutally divided into disparate camps

*R. W. Jones, *Science*, 140, 3566 (1963). See also companion article by J. S. Gray.

of practical and theoretical people. More mutual respect, at least, seems to obtain between these two sides of the track. Analog models will be increasingly resorted to by both groups in studying the formidable problems of system engineering. It is getting around generally that the modeling approach may best be taken in stages. Not only should subsystems be separately modeled and carefully confirmed, but a given model need not represent all the aspects of a given subsystem or system at the same time.

Linear approximations usually represent only a crude beginning, but may be confirmed by relatively simple analysis. Nonlinear models are harder to build, and much harder to analyze, so that frequently the approach to nonlinear structures should begin with drastic approximations to the nonlinear features, which can be refined in stages as the project develops. Each step should be simple and well defined, with continual checking of the assumptions and those portions assumed to be complete, before forging ahead. The parallel development of rudimentary overall models is in order if it is understood that they should be taken with a grain of salt: they may impart some idea of the flavor of the final concoction.

Aspects of a system suitable for separate analog study will depend on the nature of the system; this is the age of broadness of system definition, extending even to all of Society. Taking such a case, one might study population density, political stability, wealth and commerce, considering these somewhat independently before they are all joined in one model. Again, the study might be from the viewpoint of transients, or cycles, or statistics (possibly introducing random perturbations from independent sources). Still further, the item of interest might be tolerance to parametric changes, transitions from one regime to another, extrapolations backward and forward in time, and so on.

Models and analogs, even as concepts, are powerful teaching means. Symbols, words, and language are analogs right down to the ground. Physicists think and talk in models, some of the very best of them saying that models are their most powerful tools. The unification of a set of properties in one structure, suggestive of an underlying organization and beauty, gives power and appeal to the model concept in the education of students; and students we all should remain.

Emerging many years ago from the old Jefferson Physical Laboratory at Harvard, one could read on the Music Building opposite, cut into the stone under the eaves, an inscription which should still be there:

"To Charm, to Strengthen, and to Teach,
These are the Three Great Chords of Truth."