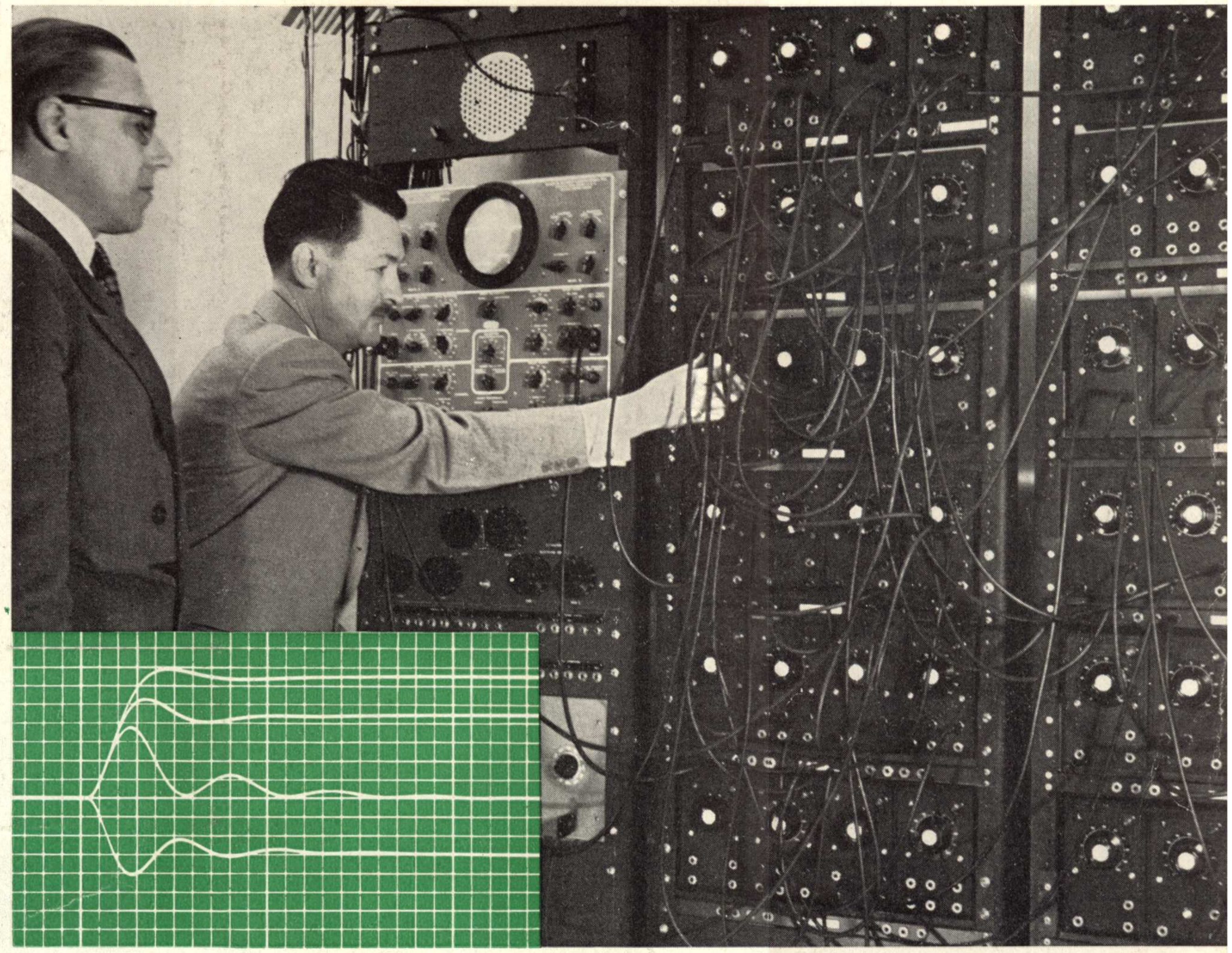
ROUTING LIST

May • 1952

industrial laboratories®

the news magazine of research • product and process development • engineering



Analog computor assembled from components solves governing problems for all types of prime movers—knob turning simulates service conditions, at Woodward Governor Co., Rockford, Ill.

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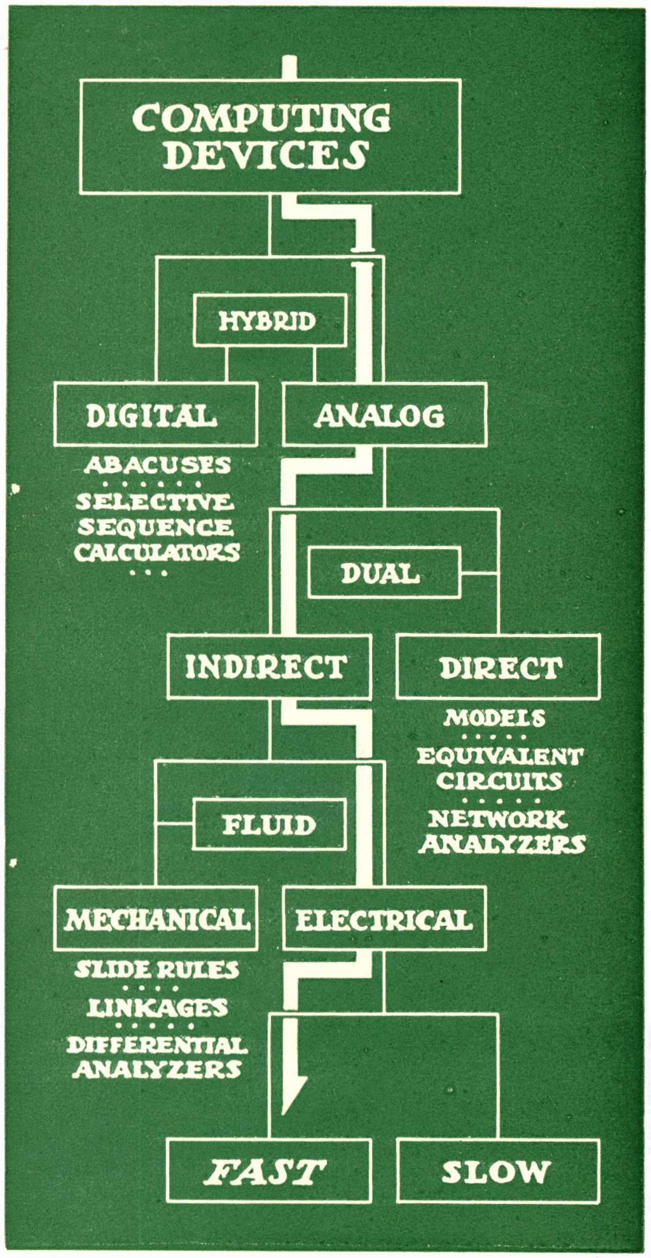
Printed in U.S.A.



The Electronic Analog

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Would you drive a spike with your fist? No? Then why bruise your brain on an armor-plated differential equation? Man is a user of *tools,* and is helpless without them. His aboriginal club and spear outdid the beast; and now the superiority of his implements measures the material advance of civilization. All *instruments,* however elaborate and refined, are just tools along with hoes and hammers. They extend the power of our muscles, the length of our arms, and the sharpness and range of our perceptions. At the upper stages, culturally, come the tools which extend the powers of the intellect. And chief among these we count computing devices, or *Computors.*

The computing machine is becoming as essential to our modern existence as electricity and the automobile. Already with the help of computors, innovations have been wrought that would assuredly have been unthinkable without their use; recent rapid advances in the fields of nuclear science, supersonic flight and rocketry offer strong testimony to their valve in analysis, design, development.

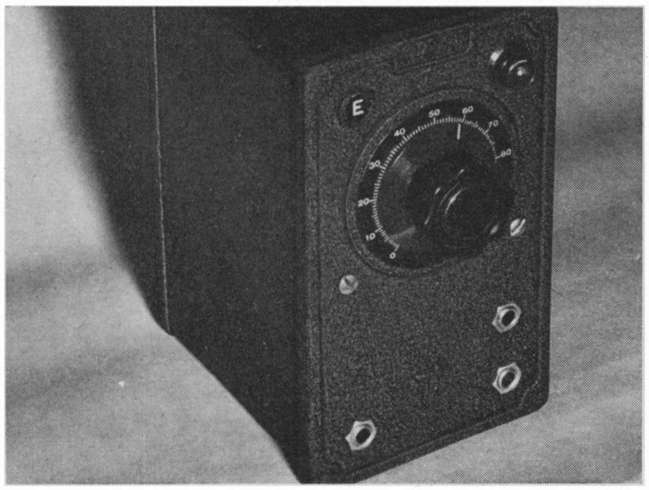
Not only have computors been used to obtain increased knowledge and more accurate, rapid prediction of the behavior of physical processes, but on an ever greater scale computing equipment is being incorporated as an essential element in the control itself of such processes. Present examples of such applications are the well known computing gunsights and other fire-control equipment, as well as the less well known but equally successful uses of computors to operate machine tools, and for continuous automatic process and quality control. Potential developments of this sort seem unlimited.

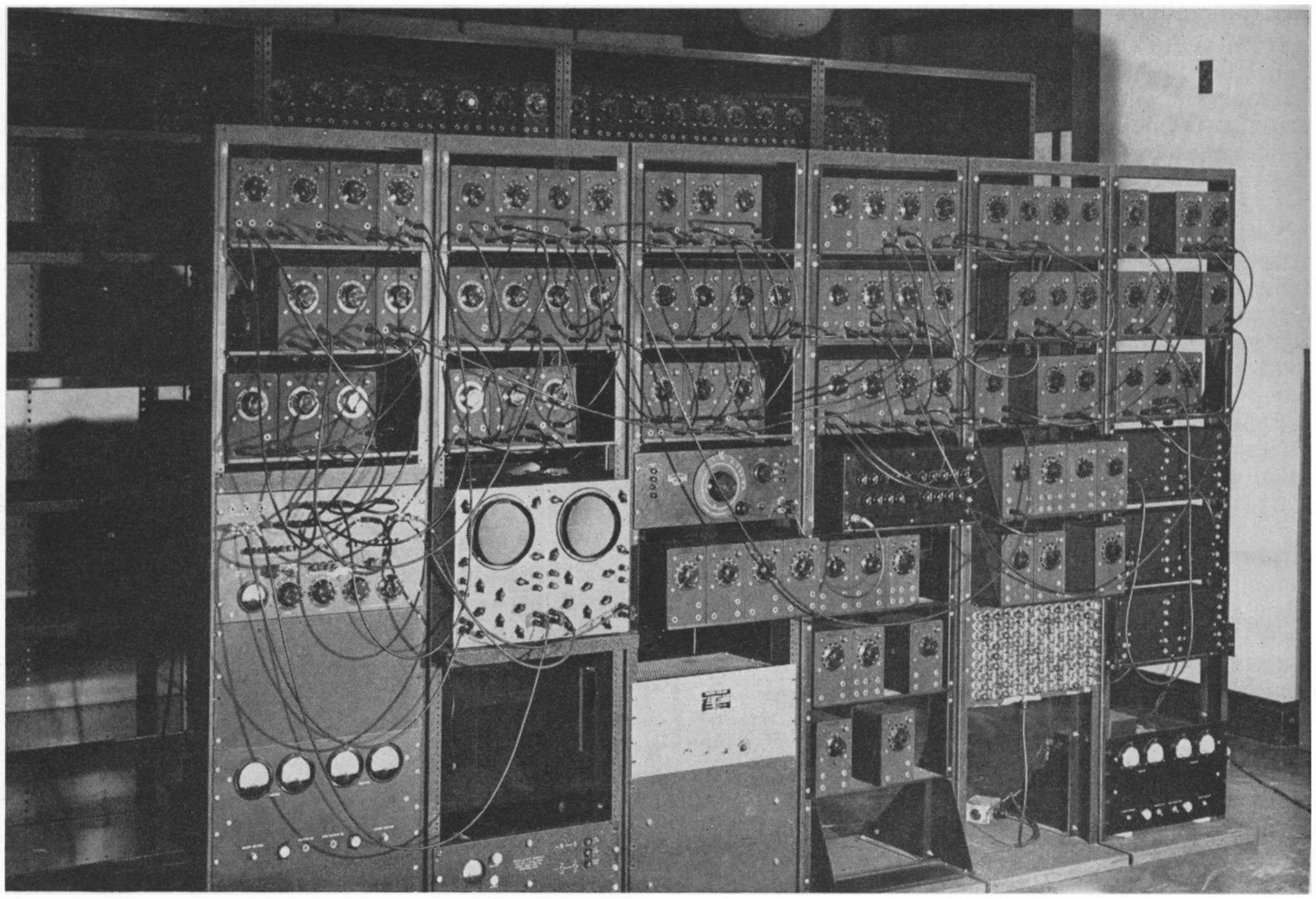
Digital and Analog

Computors come in all sizes, all speeds, all prices. Here we shall concentrate on the *fast electronic analog,* but first we must place this type properly in the hierarchy of all computors. The two major categories, nowadays, are called *digital and analogical.* The one deals in numbers only, the other in continuous physical variables. The first computes by repeatedly refining an approximation, and its accuracy is potentially unlimited; in the second type some sort of physical model is set in operation to generate a solution which is valid by virtue of an analogy to the problem at hand. This latter type, the analog computor, is limited in accuracy by the physical elements of the model.

It is frequently constructive to employ more than one type of computor on any given problem, since

Above: skeletal representation of computor classes, with a few examples. Below: typical electronic analog computor component. Input signal enters via jack at the left; outputs of either sign are taken from right-hand jacks. Cut at right shows assembled computor used in Philbrick Researches' home lab for design problems in circuits, feedback controls, and for consulting work.



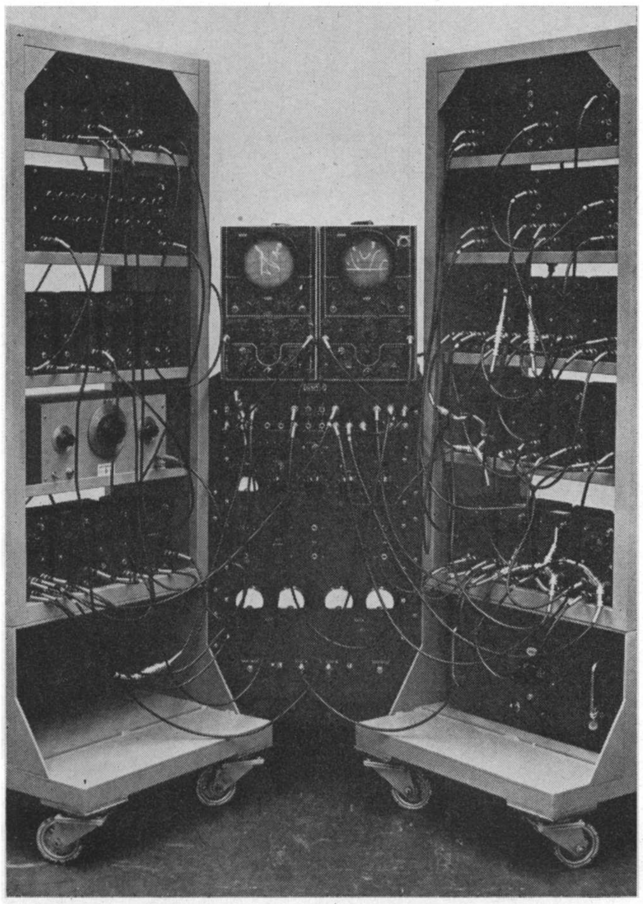


Computor as a Lab Tool

each type may be most effective for certain phases. Thus analog and digital methods may complement one another when applied in sequence, the first pointing the way to an answer and the second leading to any desired precision of the final results. As to all the types and combinations of computors which are in use and in development, there is no pretense here of complete generality or inclusiveness. Space limitations, alone forbid such scope.

We leave the digital types now (to their proponents), and proceed in more detail to break down the analog category into its several branches. One important branch, not usually called "computors," is represented by true *scale models*. In these a structure or phenomenon is reproduced by changing its physical dimensions without discarding the general physical form. To this branch belong such models as the tiny experimental airplane in a wind tunnel, the miniature dam and dammed lake on a table top, and even model transmission lines reduced to laboratory size. Somewhat less direct models are those in which the medium is altered: for example the electrolytic tank used in the study of hydromechanical field problems. Still less direct, but models nevertheless, are the semi-mathematical types normally identified as analog computors.

Analog computor used regularly in an industrial laboratory of a large chemical compnay for sutdy of control problems, analysis of reaction kinetics, and solving nonlinear differential equations.



The physical elements of such computors make available a set of *operations*, which may be chosen either because they are mathematically fundamental, or because they recur as dynamic relations in the structures being studied.

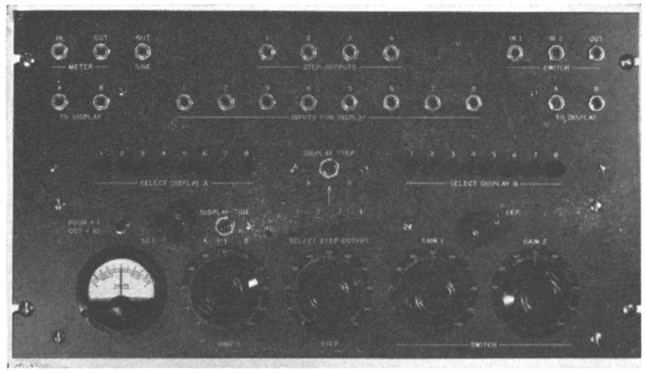
Various Media

Indirect analog computors generally carry out or assist in the solution of equations, algebraic and differential, linear and nonlinear. An extremely familiar and simple example is the *slide rule*, which is entirely algebraic. At the other extreme of elaboration is the *differential analyzer*, which may be a roomful in itself, and which can take some formidable differential equations in its stride. Actually this general class of computor is represented in many physical media, even including pneumatic and hydraulic; the predominant realms, however, are the mechanical and electrical. The differential analyzer already mentioned, being based on a mechanical disk-type integrator mechanism, belongs to the mechanical group. This instrument is very far from being outmoded, but as a practical matter it cannot be operated at a conveniently high speed for certain applications.

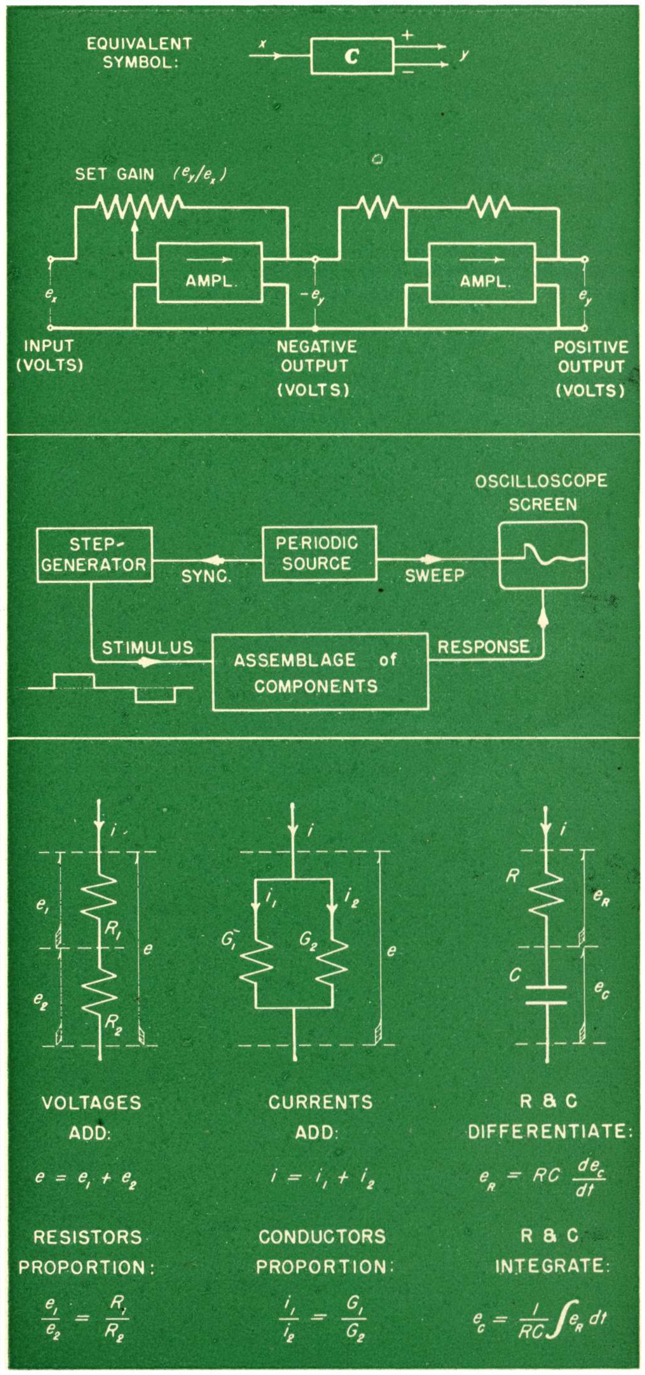
We turn now to the *electrical* (and electronic) *analog computors*. Some machines in this class are often referred to as "differential analyzers," but this is perhaps an impropriety in view of the connotations of that term.

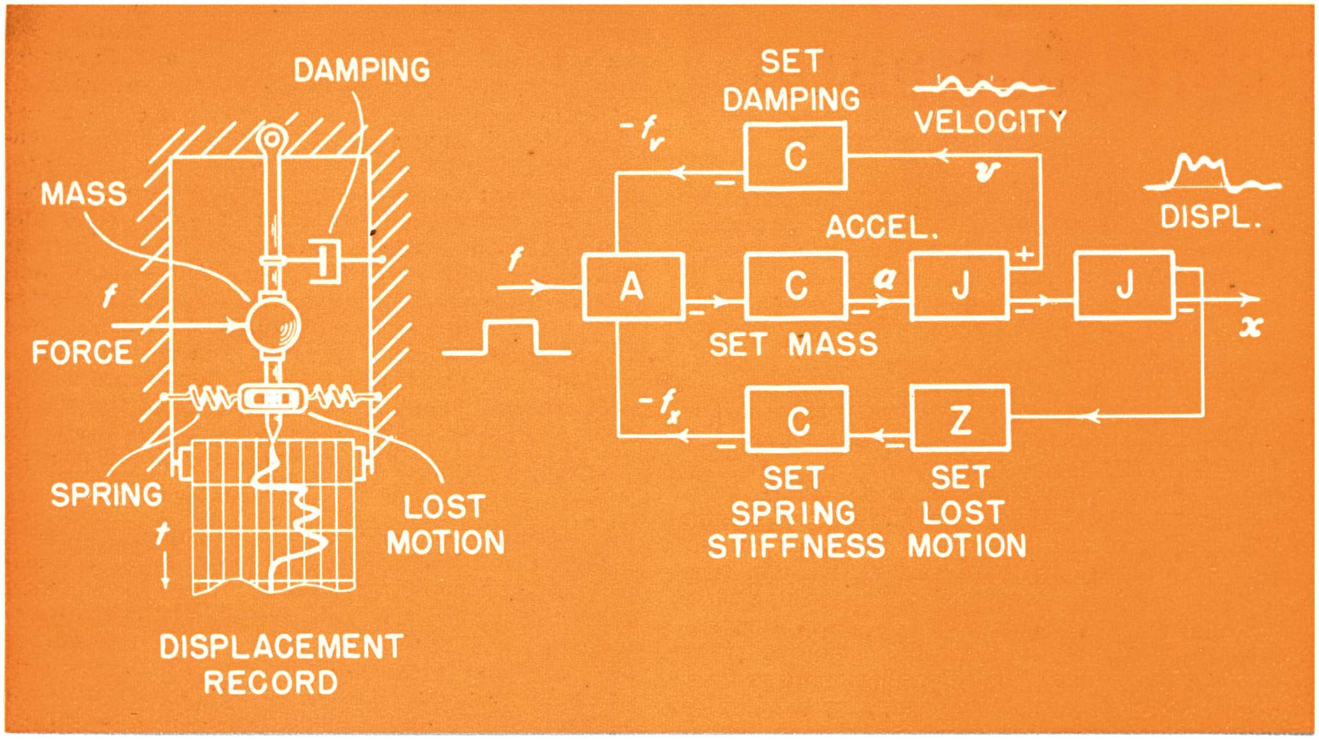
Electrical analogs of the indirect class normally employ high-gain amplifiers, which are applied in feedback loops to assure accuracy in terms of a few highly consistent circuit elements. By such means it is possible to take advantage of the precision and other desirable properties of those elements, and accordingly to perform quite strict mathematical operations among the variables in an artificial system. Let us take a look at some fundamentals, beginning with purely conductive or resistive elements. If such elements are arranged in series, the voltage across an entire set is equal to the sum of the voltages across the individual elements. Again, if conductors converge, or branch from a single conducting element, the current in the single element is precisely the algebraic sum of the currents in the branches. Either of these physical truths may be applied to make a reliable *adder*. Look further at the property of resistance, or at the inverse property of conductance. In any given electrical path, these properties fix the proportionality between current and voltage in a precise way. For example if a pair of resistors have equal currents, the voltages they experience are in proportion to their resistances. In an appropriate feedback loop, this fact may be used to magnify or diminish a computed signal in a ratio which may be accurately determined and adjusted. Thus for example one can "multiply by a constant" of any magnitude, or multiply voltages by each other.

Another electric circuit element available in pure form is the capacitor, which is able to store a charge. The charge it stores is equal to the product



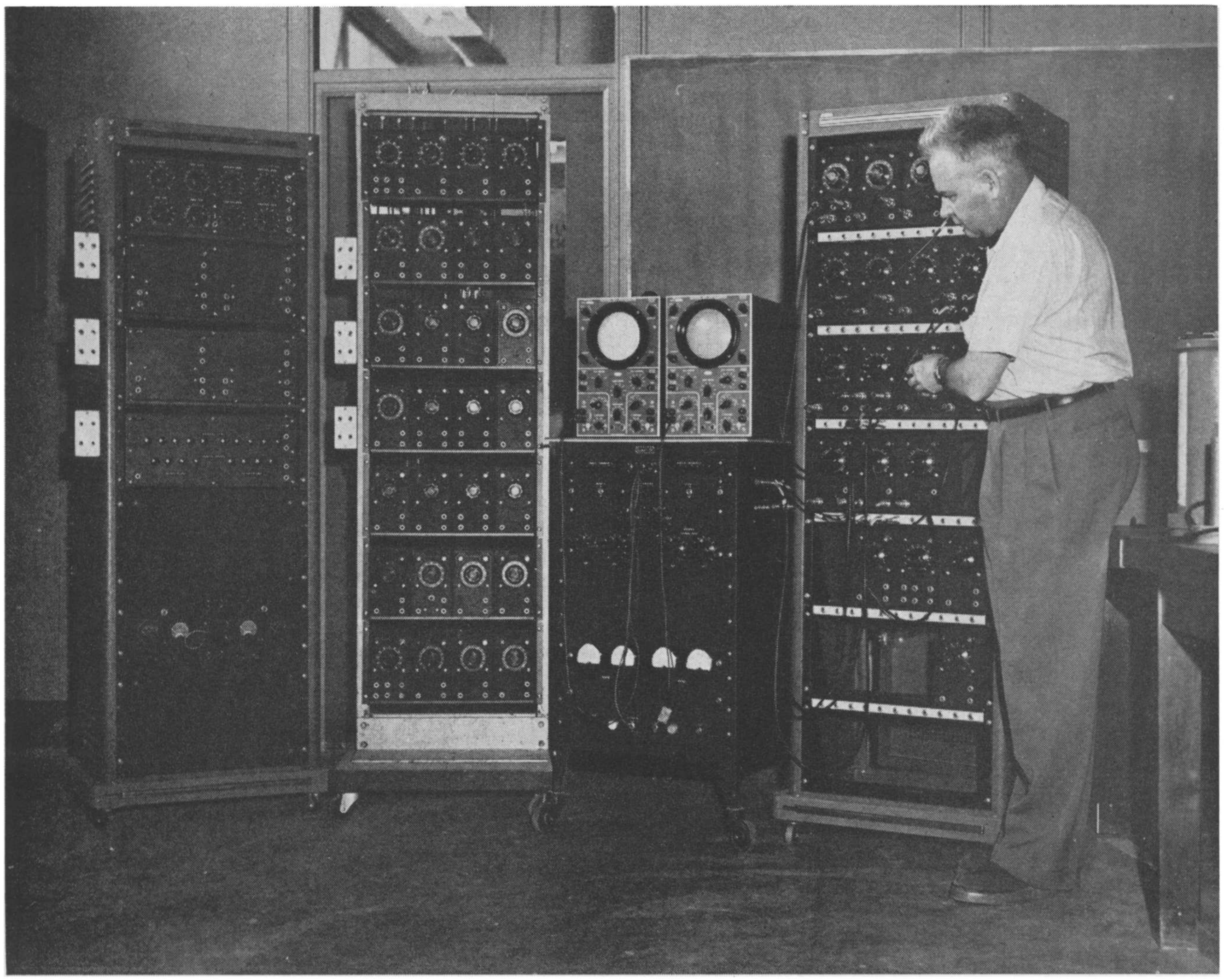
Above: "central component" used as convenience with assemblages to provide initiating signal, enable facile connection to scopes. Below (top down): example of component with simplified graphical symbol; interconnected assembly forming computor; properties in addition, proportion, differentiation, integration.





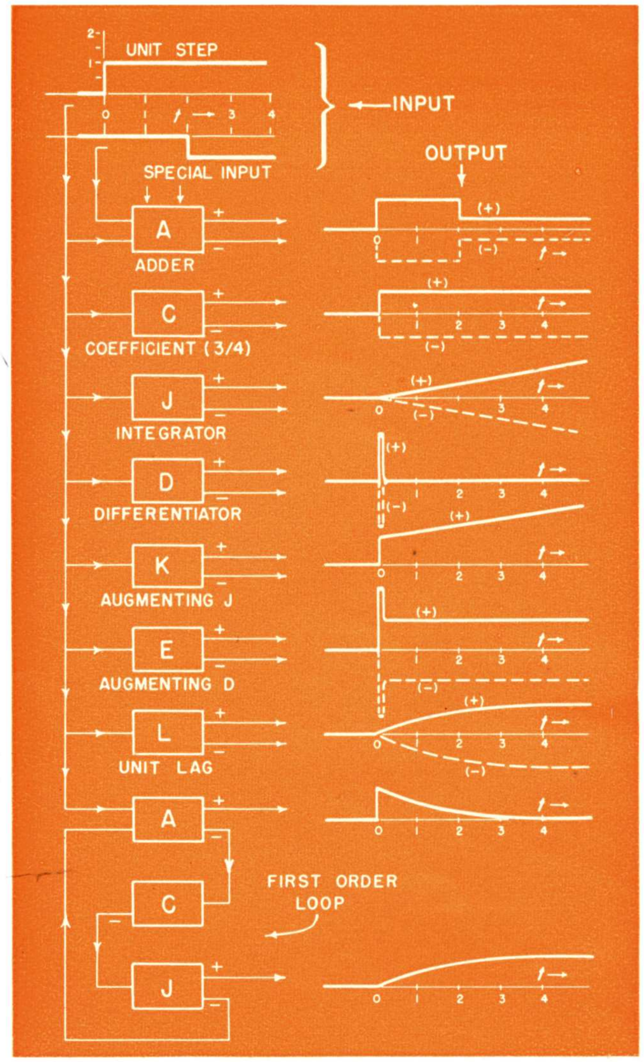
▼ Class demonstration set-up for specific servomechanism problem in Department of Engineering, University of California. Equipment is used for instruction and advanced student projects.

Above: simple mechancial system and analogous arrangement of computing components. Lower right: names, symbols, transient responses, simple combination for basic linear operations.



of its capacitance and the voltage across its terminals. Thus the charge is measurable by means of this voltage, and being quantity of electricity its rate of change is simply the current supplied to the capacitor element. Conversely, the voltage is proportional to the time integral of the incoming current, and the proportionality is directly dictated by the value of the capacitance. These facts lead to a means for *integrating* and *differentiating* with respect to time.

It is convenient, however, instead of dealing with both currents and voltages, to employ one or the other type of variable alone for the inputs and outputs of computing operations. By applying regulatory—or *feedback*— techniques, the above elementary circuit arrangements may be adapted in such a way that the relevant operations take place between applied currents and responding currents, or between applied voltages and responding voltages. Either currents or voltages may serve in practice as computing signals: the choice is up to the designer. It happens that voltages, applied and measured with respect to a common conductor (or "ground"), have become most popular. In any event, one is led by these techniques to devices in which the operations of addition, multiplication by adjustable factors, and differentiation or integration with respect to time are automatically performed between inputs and outputs; and generally these latter are voltages with respect to a con-

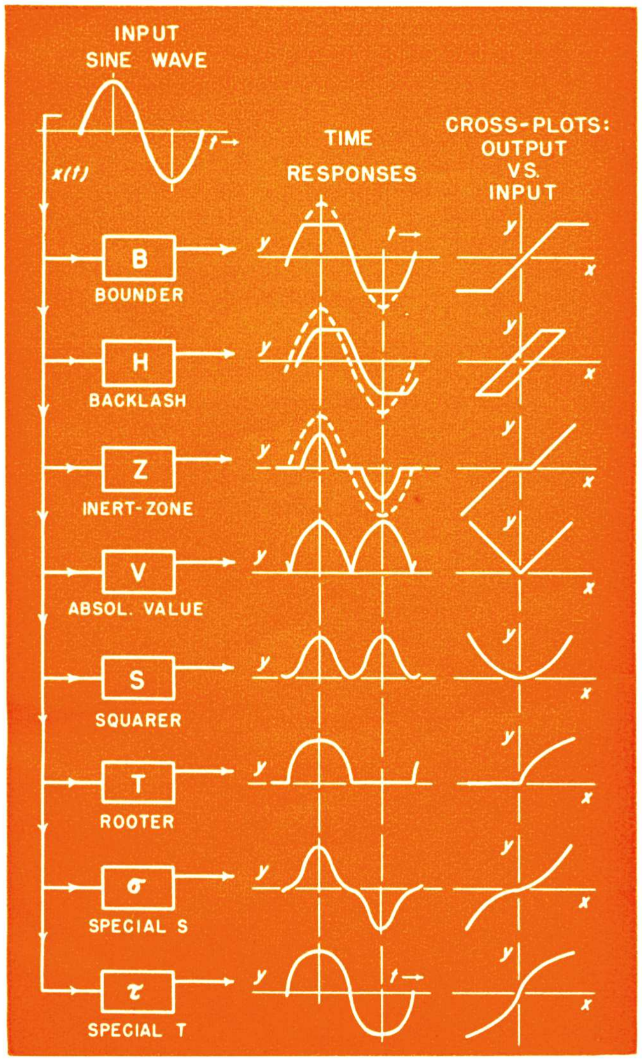


The Electronic Analog Computor

venient reference. In a computing or analog assembly, of course, these voltages correspond to the *variables* of the equations or systems represented.

Combining of Operations

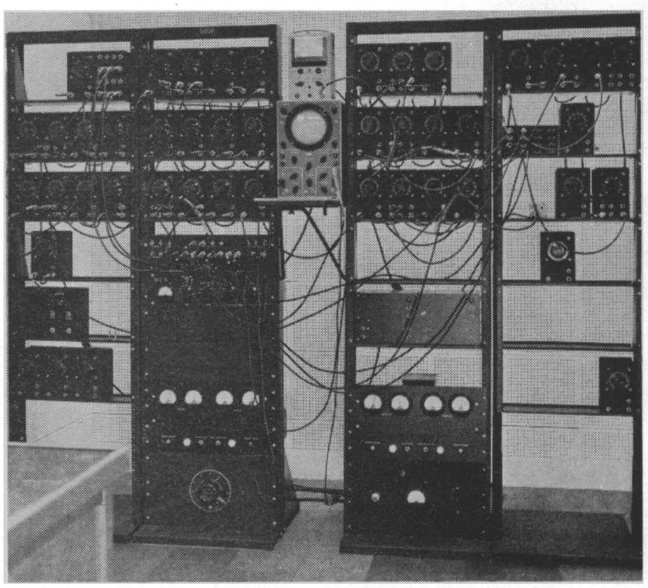
Thus far it has been shown how to *add*, to *proportion*, to *integrate* and *differentiate*, and how such operations may be embodied in individual instruments (or components) to serve as building blocks for fashioning a computor. It should also be pointed out how general these few operations really are. Note first that the proportioning operation will permit multiplying by negative as well as positive factors, and in particular by minus one. This enables the extension of adding, for example, to include the operation of *subtracting*. These two operations, in combination with proportioning to establish coefficients, and with full freedom of repetition and sequencing, will be seen able to reproduce and represent any first-degree arithmetic operation, and hence may be applied to "solve"sets of linear algebraic equations. The technique is



simply to interconnect the operational components as dictated by the equations to be solved. Solution is then performed by the assemblage with no further outside ministrations, provided the power is on and all connections are properly established. One reads off the answer by measurement of the voltages corresponding to the unknown variables.

Usually more than one arrangement is possible for satisfactory solving a given problem with operational components such as those described above. But whatever the arrangement, certain rules must be observed. Of course it is evident that the known variables are fed in from without, and that the known or assumed characteristics are set in by adjustment of the various components. In general there will be several unknown variables involved, and there may be a choice as to how the corresponding voltages in the computor are allowed to affect one another. In these relations we recognize a *casual* property. For any component of the above type there is a definite direction from input to output, coinciding with the direction from cause to effect. This implies a *casual order* among the variables in the problem as set up on the computor, an order which is not explicit in the mathematical formulation. Thus there is some room for imagination in setting up a computor, and often there are many approaches leading to success. We cannot cover all the techniques of computing in this space, but it may be pointed out at least that in general, *loops* of casual activity are formed in all but the simplest cases. As a result the question of stability arises, as with casual loops in other connections, and much attention is given this subject by those who design and deal with such instruments. Suffice it to say that stability may always be achieved.

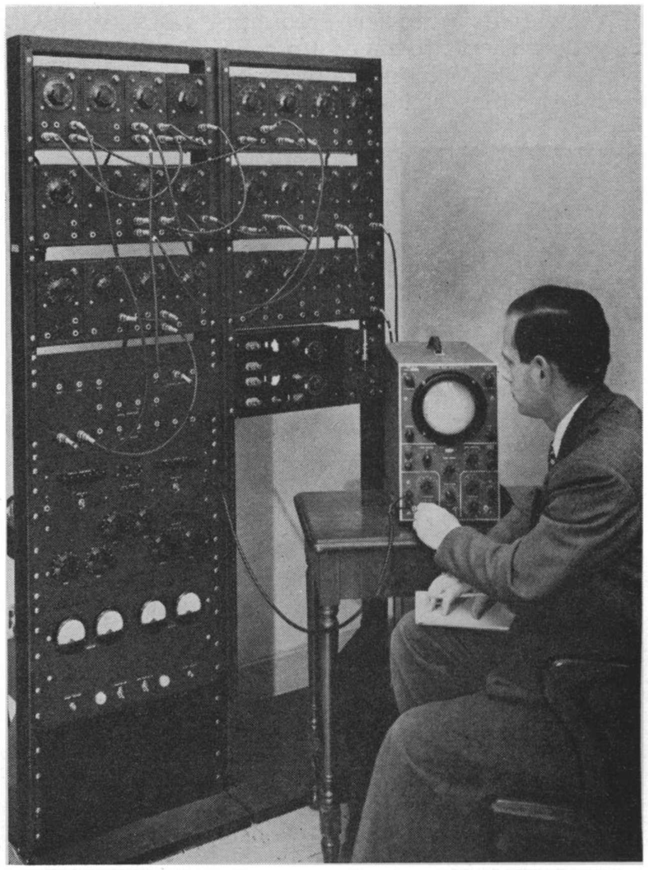
Left: components and characteristics for basic non linear operations—positive outputs only. Below: turbine control lab installation of Pratt & Whitney Aircraft, East Hartford, Conn. Right: high speed unit used by the Shell Oil Co., Emeryville, Cal.

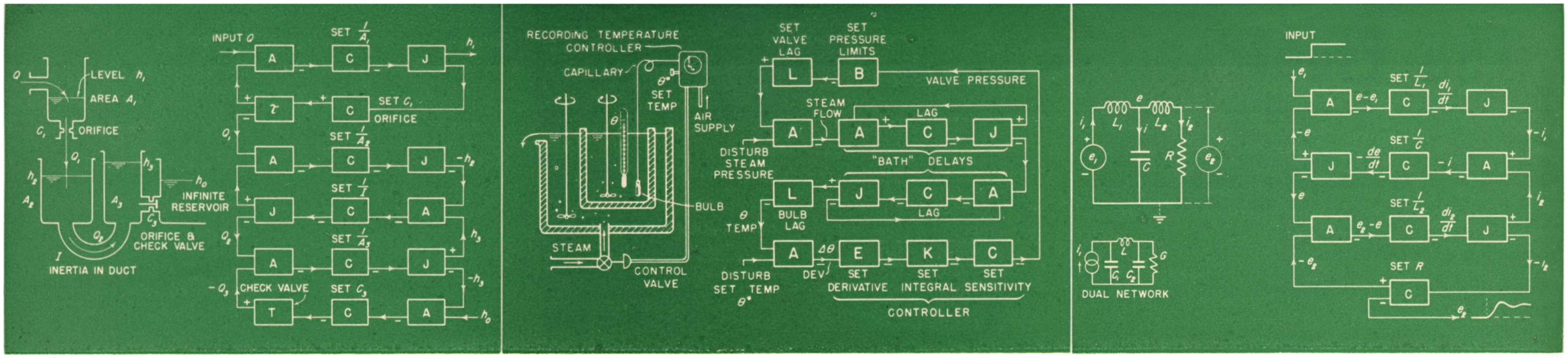


When time and rate of change are added as variables, algebraic problems turn into *calculus*. We have, for example *differential equations*. The addition of either differentiating or integrating components enables the computor to handle the linear class of such equations. It has turned out, for reasons of stability as already mentioned, that integrating components are usually more practical, but it is not uncommon to employ both types in the same problem. At least ideally one type (the integrator say) will cover all cases in the solution and study of linear phenomena, and any linear system may be represented by combinations of the three components for adding, proportioning, and integrating. Parenthetically: for evident reasons, the proportioning component is often called the *coefficient* component. A bit later we shall discuss techniques and equipment for nonlinear applications.

Slow or Fast?

The various electric analog computors discussed to this point have not been identified as to speed of operation. They may be designed to operate exceedingly fast, completing a solution in one microsecond or less. They may also be built to run very slowly indeed, taking days to "figure out" just one problem. An optimum speed, between these limits, is generally dictated by the economics of the application. Sometimes a computor of this sort is employed to *simulate* the operation of some portion of a system or process under development, and then its speed is automatically determined by that of the part being simulated. When there are no criteria of this sort present, there is a very important advantage in choosing a speed of operation such that a solution is completed faster than we can change the conditions and characteristics of the





▲ **From left to right: a thermal system under temperature control and its analog representation; nonlinear hydraulic system in the transient condition and the corresponding analog; electric filter network, analog embodying dual network.** **(See page 13.)**

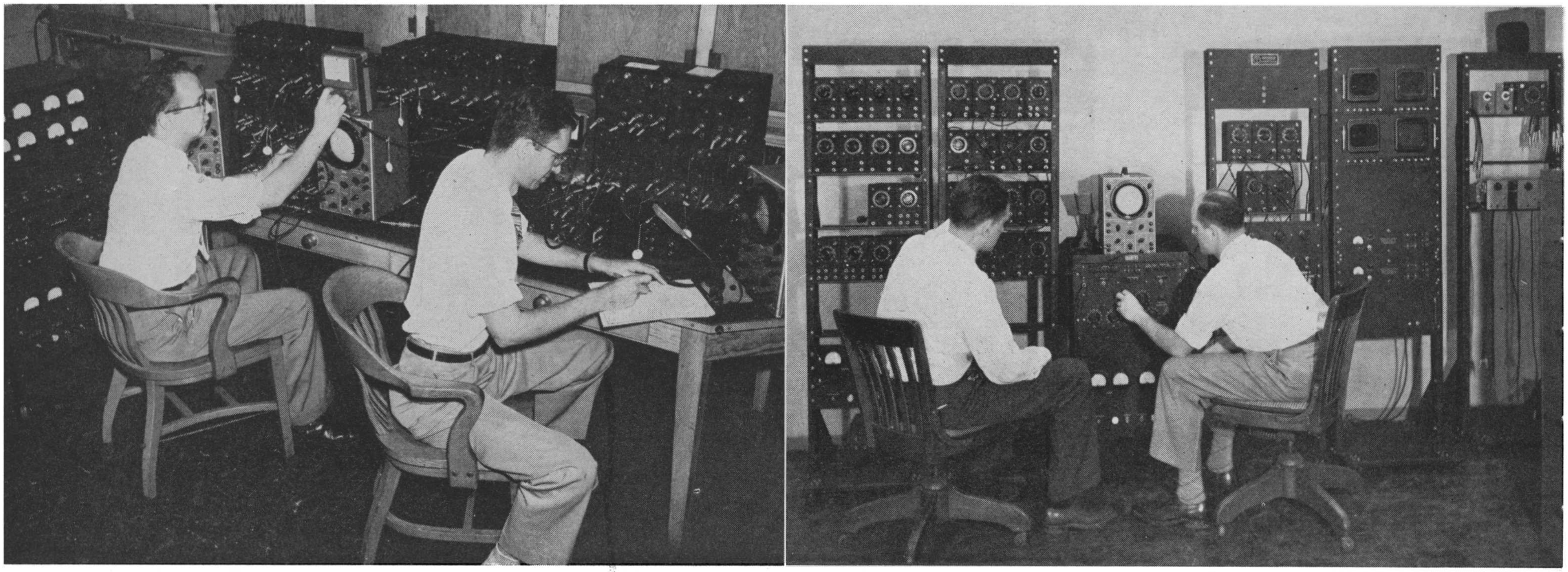
problem. This is true whether the equations are algebraic or differential. In the latter case, where time is the independent variable, a fast computor enables repetition of the solution with a period so short that a sustained oscillographic display of the unknown variables is possible, showing the solutions continually to the operator as apparently stationary functions on the screen. The screen pattern, of course, alters as the initial conditions or the parameters are manually changed. No waiting period is required to study the solutions for a wide variety of cases.

Fortunately the cost of an electric analog computor is minimum for this reasonably high-speed version. Electronic techniques and manufactured parts are plentiful for use in the audio-video band which is involved in devices of such a time scale. In terms of frequency, it is possible to employ a band extending from the repetition rate as fundamental up to frequency a thousand or more times higher. The usual procedure, however, is to operate all the way down to zero frequency (DC). While it may be difficult to speed up a DC Computor, it is relatively easy to make it operate more slowly merely by increasing the reactive impedances.

Electric analog components which add, set coefficients, integrate and differentiate have already been referred to. As a practical matter, these are designed with high-impedance inputs and low-impedance outputs, for foolproof interconnection in a computing system. Algebraic sign is allowed for by providing both positive and negative outputs for each component. With these and a few other practical features — for supplying power, adjusting, identifying, and so on—such components are engineered for convenient usage as versatile general-purpose analog *building blocks.*

While the above component operations are sufficient for all linear mathematics, or model-building, it is economical to add certain other components in the linear class for combined operations which keep arising. A large number of such combinations are current, some fairly complex; we mention here only three of the simpler examples. One is like the integrator, but transmits an additive signal equal to the input: this component is called the *augmenting integrator*. Similar in concept is the *augmenting differentiator*. An accompanying illustration shows responses of these and other components when subject to a test square wave. These augmenting variants are particularly useful in the study of control problems. Another such component is that for the simplest dynamic lag. Called the *unit-lag* component, it actually operates inversely to the augmenting differentiator. These two in sequence will leave a signal unchanged, if the adjustments are matched.

In this box of tricks are also components which embody basic nonlinear properties, notably *bounding, backlash, inert-zone, absolute-value, squaring,* and *square-rooting* components (variants of the last two are included which feature a complete inverse symmetry useful especially for hydraulic problems). All these characteristics may be demonstrated in terms of their response to a sine wave input, plotted either against time or versus the in-



▼ **At left: equipment applied by Curtiss-Wright Corporation, Woodridge, N. J., to computing histories of several variable during trajectories of jet missile. Right: installation used in industrial control engineering by Askania Regulator Co., Chicago.**

put itself. Components in this group are employed to represent real physical discontinuities and "distortions" which are often assumed absent in conventional analysis. In many cases the effects of such irregularities are important, often crucial.

Only an idea may be given here of the scope of practical applications of the computing apparatus described. The broad mathematical usages should be evident from the text; an exhaustive summary along this line would depart from the matter of the present exposition, and might even be misleading. It is preferable to pick out a few familiar physical systems and to show how analog computor components may be interconnected to embody the laws and to represent the phenomena which operate therein: and thus to illustrate how dynamic performance may be "solved for."

A first example is a mechanical structure exhibiting damped vibrations, involving a massive body, a spring, and a dashpot. Here the position of the body is developed as the integral of its velocity, which in turn is the integral of acceleration, which itself is related to the net force by the magnitude of the mass. The net force, finally, is the algebraic sum of an applied or external force, a spring force depending on position, and a damping force which depends on the velocity. Nonlinearities such as lost motion may be incorporated directly. All physical properties are quantitatively adjustable in the computing assembly.

Another example is offered by a thermal system under automatic control. The temperature in the inner of two concentric baths is to be regulated at an assignable value. Transfer lags are interposed between the application of heat and the location of the thermometer. Lagged responses are also present in the steam valve and in the bulb itself. One may represent, for example, all the classical control reactions, integral, proportional, derivative, and so on which might be applied in the regulatory installation. Limits of valve operation are directly imposed by the nonlinear bounding component. Solutions for optimum recovery, and consequently the most stable and effective controls, are obtained in short order by experimental variation of the design parameters under appropriate disturbances.

Seemingly paradoxical is the application of electronic computors to electrical networks, which are themselves used as models for phenomena in other media. Curiously, this is one of the most powerful and penetrating applications of the computor. The relations between voltage and current, for each inductor, capacitor, and resistor in the network, are merely set up in a consistent casual sequence by means of the simplest standard components. The result is a computing structure of wide flexibility whereby the desired network may be rapidly designed. Dual networks are implicit in the assembly: one needs only to reverse the roles of voltage and current. Negative values for each circuit element are as easily set in as are positive values.

Finally a hydraulic example may be selected, involving storage tanks, the inertia of fluid in a duct, nonlinear resistance to flow, and a check valve. The variables are levels (or hydraulic *heads*) and flows. The check valve may connect one tank to an infinite reservoir at fixed head.

These examples demonstrate that a broad variety of useful systems and operating structures may be synthesized and analyzed to any desired detail with analog components of the type described.