

AN INTRODUCTION TO Analog Computors BY JERRY ROEDEL

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AN INTRODUCTION TO ANALOG COMPUTORS

ABSTRACT

As an introduction, to the newcomer, into the field of computing devices the history and classification of these devices is duscussed with particular emphasis on analog computors. A discussion of the precision of computing devices follows to acquaint the reader with this important problem. The paper is concluded with a consideration of basic elements in the operational amplifier type of analog computor, and is illustrated with photographs of typical installations of this type.

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INTRODUCTION

Although this paper contributes nothing new to the art of computing devices, the need for a very general introduction to this subject is well recognized. It is for the benefit of the neophyte that this paper will discuss the history and classification of computing devices. To introduce more detailed concepts of analog computation, a discussion will follow of the basic elements of the operational amplifier analog computor. It is with this equipment that the author is most familiar.

HISTORY

The history of computing devices may well extend to the very beginning of civilization. The early forms of digital computation probably manifested themselves when early man first started to keep tab on numbers by counting on his fingers or with pebbles. The earliest known record of analog computation is the surveying and map making in Babylonia in 3800 B. C. for the purpose of taxation. By 1300 B. C. surveying and map making were common in Egypt.

The earliest digital machine is probably the abacus which evolved from the counting with pebbles. In its early form, which was used in the Tigress Euphrates Valley as early as 5000 years ago ³ and in Egypt as early as 460 B. C., it consisted of a clay board with groves in which pebbles were placed. It later appeared in Rome, China and Japan in the form of a wire frame with beads. A picture of the Japanese abacus (Soroban), which is still used by Japanese and Chinese tradesmen, is shown in Figure 1.

In 960 A. D., Gerbert brought back from the Universities of the Moors the concept of Arabic numerals, and tried for years to make practical a calculating machine, which the Moors had conceived. John Napier described his invention of logarithms in 1614 and in 1615 John Briggs, in collaboration with Napier, converted them to the base 10. In 1617 John Napier devised a method of multiplication utilizing numbering rods. These were known as "Napier's Bones." Edmund Gunter utilized Napier's logarithms in 1620 to create a slide rule with no moving parts. This was subsequently improved on by William Oughtred's conception in 1632 of the astrolabe, which was the forerunner of the slide-rule and nomogram with a sliding scale.

Blaise Pascal in 1642 invented the first desk calculator. This device, which is shown in Figure 2, using toothed wheels was limited in its operations to addition and subtraction.

G. W. Liebnitz made an important contribution by his improvements to the Pascal machine in 1671, 10 to facilitate multiplication by repeated addition. His first machine, Figure 3, was completed in 1694, but it was never practical because of mechanical imperfections.

The planimeter, an analog device first appeared in 1814 after its invention by J. A. Hermann, a Bavarian engineer. Between 1814 and 1854 when Amsler invented the popular modern polar planimeter, Figure 4, many new and improved types of planimeters appeared.



FIG. 1 A JAPANESE ABACUS of the type used today by Japanese Tradesmen



FIG. 2 ONE OF PASCAL'S CALCULATING MACHINES Courtesy of Felt & Tarrant Mfg. Co.

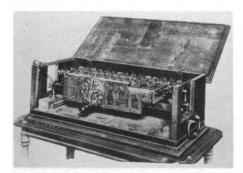


FIG. 3 LEIBNITZ CALCULATOR - The first two-motion machine designed to compute multiplication by repeated addition. Courtesy Felt & Tarrant Mfg. Co.

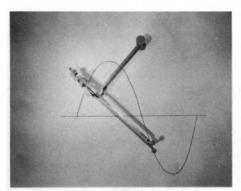


FIG. 4 POLAR PLANIMETER -Based on Amsler's invention in 1854

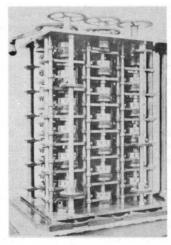


FIG. 5 BABBAGE'S DIFFERENCE ENGINE - Mr. Turck states that this part consists merely of an accumulator mechanism Courtesy of Science Museum, South Kensington, England

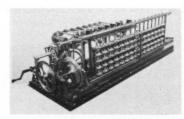


FIG. 6 SCHEUTZ DIFFERENTIAL CALCULATOR Courtesy Felt & Tarrant Mfg. Co.

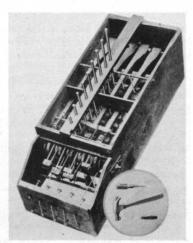


FIG. 7 FELT'S "MACARONI BOX"- This is the first model of the comptometer which Felt built using the tools shown in the insert. Courtesy Felt & Tarrant Mfg. Co.

Charles Babbage in 1812 conceived a digital difference-equation solver which was to be complete with means to print the answers. The English Government supported the construction of this machine until 1833 when work was suspended, and they finally abandoned the project in 1842. I have been told by Mr. J. A. V. Turck that his inspection of the only part of this machine which was ever built, shown in Figure 5, revealed it was merely an accumulator mechanism.

Thomas De Colmar, in 1820, made improvements on Pascal's calculator. This machine, which was considered to be the first successful machine for multiplication, is still made in Paris by Darras. Thomas himself made 1500 units of his 6 place machine by 1878.

It was in 1833 that Babbage conceived his analytical engine. This was to be a more versatile machine than his earlier unsuccessful "Difference Engine". It is said that this machine was the true fore-runner of the modern large-scale digital computers. The memory and programming were to be in the form of Jacquard-Hollerith principle cards while the arithmetic section used tooth wheels.

Scheutz in Sweden completed a machine based on Babbage's idea of the "Difference Engine" which he exhibited in London in 1855. The original model, shown in Figure 6, is now in the museum of the Felt and Tarrant Mfg. Company.

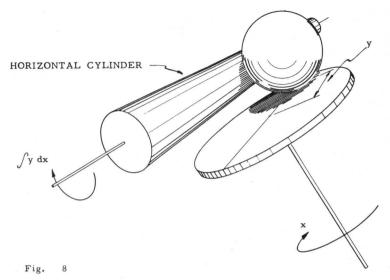
The patenting of the first key-driven adding machine in 1850¹⁸ was followed by thirty-six years of activity before Dorr E. Felt developed the first practical key-driven adding machine in 1886¹⁹ A photo of the wooden box model and the tools used to make it are shown in Figure 7.

The early art of analog devices was active in the time of Lord Kelvin. Kelvin's brother, James Thomson, had invented an integrating mechanism, shown in Figure 8. Lord Kelvin conceived the idea of connecting these devices together to solve differential equations in 1876, and an early use of Thomson's integrators is recorded in Kelvin's "Harmonic Synthesizer" built in 1878 to predict tides. 21

Leon Bolee introduced in 1887 the first calculating machine to have single operation multiplication. A model of this machine, shown in Figure 9, is in the museum of the Felt and Tarrant Mfg. Company.

Many patents were issued for improvements to adding machines following D. E. Felt's patent in 1887 of the "Comptometer". Mr. Turck states that D. E. Felt's addition of the printing feature to the Comptometer in 1889 ("comptograph") was the first practical printing adding machine. The Monroe and Marchant Calculating Machines, shown in Figures 10 and 11 respectively, were introduced about 1911. By 1920 electric motor drives were incorporated into calculating machines.

The network analyzers for the simulation of power networks appear principally as developments of the General Electric Company and the Westinghouse Company. The D. C. Network Analyzer shown in Figure 12 was the first form of these devices to appear in 1925. Since this is a resistive analog



ARTISTS SKETCH OF THOMSON'S INTEGRATING MECHANISM



FIG. 9 BOLLEE CALCULATING MACHINE Courtesy Felt & Tarrant Mfg. Co.



FIG. 11 EARLY MARCHANT CALCULATING MACHINE



FIG. 10 EARLY MONROE CALCULATING MACHINE

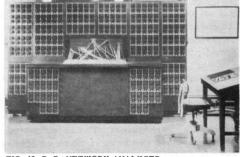


FIG. 12 D.C. NETWORK ANALYZER

Courtesy American Gas and Electric Service Co.

it is limited to steady-state problems which are either purely resistive or purely reactive. The A. C. Network Analyzer, shown in Figure 13, which was introduced in 1929²⁶was a much more versatile machine. It included three types types of linear impedances, which made it usable for the simulation of alternating current power networks showing both phase and mgnitude. The most recent of the network analyzers is the Transient Network Analyzer which is a development of the General Electric Company. Component values are readily changed on this machine and non-linear elements can be simulated. This flexibility makes this machine a true general purpose computor.

Mechanical integrating devices were improved on when Hannibal Ford during World War I increased the torque output of the ball-and-disc integrator to make a naval gun-fire computor. This was followed by more experimentation in the 1920's at M. I. T. which led to Dr. Vannevar Bush's completion in 1931²⁷ of the first large scale mechanical differential analyzer. This machine, shown in Figure 14, originally built at M.I. T. is now installed at Wayne University in Detroit, where it is still being effectively used. Dr. Bush's mechanical differential analyzer was followed by the installation of a mechanical differential analyzer at the University of Manchester in England in 1935. Further work in this field by Bush and others brought more improvements until at the present time there are several large scale mechanical machines in operation in this country.

Simultaneous equation solvers and harmonic analyzers of many types appeared in the 1930's. Among these are Wilbur's Mechanism, ²⁸ a mechanical means of solving simultaneous algebraic equations; an electrical machine developed by R. M. Mallock ²⁹ for the same purpose; an adjuster type equation solver developed by Berry et al; the Multi-harmonigraph, a mechanical harmonic analyzer developed by Brown; ³⁰ and a newer harmonic analyzer described by Hagg and Laurent. ³¹

The first large scale general purpose digital computor was completed at Harvard in 1944. This machine, the Harvard Mark I Calculator, shown in Figure 15, was built jointly by I. B. M. and Harvard. Relays were used for the arithmetic section and punched cards for read in and memory. Bell Telephone Laboratories also built a relay computor known as the BTL model I which was completed in 1940. This was a special purpose machine. Several other special purpose relay computors were built at the laboratory and in 1944 work was started on a general purpose relay computor which was to be designated as the BTL model V. This machine contained 9000 relays, 50 pieces of teletype equipment, covered 1000 square feet of floor space and weighed twice the 5 tons of the Mark I. At about the same time the Moore School of Engineering completed its all-electronic digital computor for the Aberdeen Proving Grounds. This machine, the ENIAC, shown in Figure 16, contained 18,000 vacuum tubes. It now has many direct descendants such as SWAC, SEAC, MANIAC, etc.

Although Lovell of Bell Telephone Laboratories is generally credited with the introduction of the operational amplifier during the second World War, ³⁶ such a device was independently discovered and used by Philbrick ³⁷as early as 1938 in computing circuits for the solution of servomechanism problems. The introduction of the operational amplifier has made possible the newest class of general purpose analog computor. This type is commercially known by such names as BOEING, EASE, GAP/R. GEDA, IDA, and REAC. It is also used in many special purpose computors such as those used in gun directors.



FIG. 13 A.C. NETWORK ANALYZER

Courtesy American Gas and Electric Service Co.



FIG. 15 THE HARVARD MARK I CALCULATOR Courtesy of Harvard University

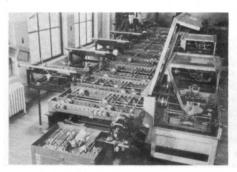
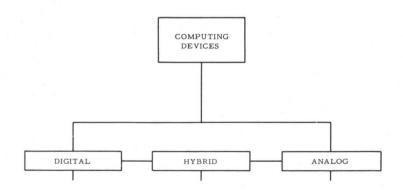


FIG. 14 MECHANICAL DIFFERENTIAL ANALYZER
Courtesy Massachusetts Institute of Technology



FIG. 16 ENIAC - The first all electronic digital computor Courtesy Aberdeen Proving Grounds



GENERAL CLASSES OF COMPUTING DEVICES

There are two major classes or categories of computing devices as is seen by the chart in Figure 17. These are, the digital computor which deals in numbers only and the analog computor which deals with continuous physical variables. With the perfection of "analog to digital" converters there should probably be a considerable swing in the direction of hybrids which are a combination of analog and digital. The digital computor works directly with integers which are expressed by gear teeth in the desk calculator or electrical pulses in the electronic digital computor. The desk calculator adds by the addition of revolutions or tenths of revolutions, while multiplication is carried out by an extension of the addition principle. The digital machine then has addition as its basic function and all other complex arithmetic is described in a logic based on the concept of addition. The power of the digital machine lies in the speed with which the machine will add. To solve the more complex problems such as the solution of differential equations, the machine computes by repeated refining of an approximation. By sacrificing speed the precision of digitally computed results can be greatly increased.

Two general subdivisions of digital computors will be considered. They are the general purpose and the special purpose machines as shown in Figure 18. In each of these subdivisions we find machines which are either mechanical, electrical or electromechanical. The only true all-mechanical general purpose machine is probably Babbage's "Analytical Engine" which was never developed beyond the stage of Babbage's original idea as to how it would work and what it would do. The ENIAC represents that group of machines in the general purpose class which is all electrical except possibly for some read-in and read-out equipment. Finally, the relay computors which are exemplified by the Harvard Mark I are predominantly electro-mechanical. A few of the devices in the special purpose class of digital computors are shown in the skeletal diagram in Figure 18.

The analog devices as has been stated operate with physical variables such as shaft rotations or electrical voltages. From the chart in Figure 19 we find two types of analogy. The direct analogy is characterized by those cases where problem variables and problem parameters are represented directly by variables and parameters on the machine. An example is the direct analogy which exists between the energy storage in a mechanical spring and the energy storage in an electrical capacitor. A direct electrical analogy for a simple spring mass system is shown in Figure 20. By application of the principle of Duality it is also possible to have a computor which operates as the dual of the problem. The mechanical direct analog computors are most generally scale models such as wind-tunnel models. The electrical direct analogs include such instruments as the network analyzers, and equivalent circuits exemplified by the Anacom type of computor shown in Figure 21. In the fluid analog are found such devices as model dams and stream beds which are found in many hydraulic laboratories. Of this group the Anacom and the Transient Network Analyzer are probably the only general purpose computors.

The indirect analog computors are of a type which can carry out or assist in the solution of algebraic or differential equations. The most

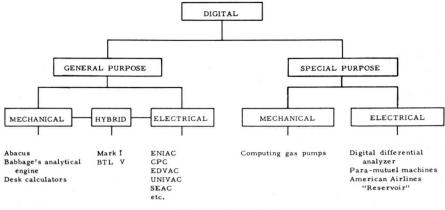


Fig. 18 CLASSIFICATION OF DIGITAL DEVICES

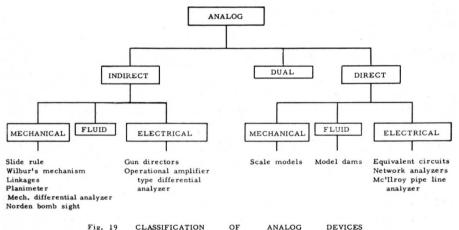


Fig. 19 CLASSIFICATION OF ANALOG

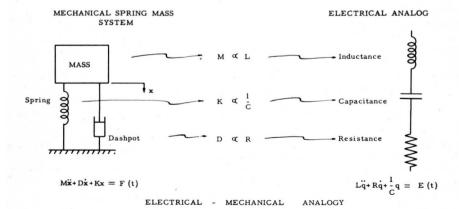


Fig. 20

common example of a mechanical indirect analog computor is the slide rule where lengths on a stick are analogous to numbers. Nomograms and various linkages are also in this category. The Norden Bomb Sight is a mechanical analog computor of the special purpose variety. The true general purpose indirect mechanical analog computor is the mechanical differential analyzer which Lord Kelvin wrote of and which Vannevar Bush and others have subsequently made practical.

The fluid type of indirect analog computor is the type which is least common. This is presumably because of the difficulty in measurement in the fluid system.

The electrical (or electronic) indirect analog computor is probably the most common of the indirect type. This type normally employs high gain amplifiers which when applied in appropriate feedback loops will perform many mathematical operations. It is the computors in this class which use the operational amplifier discovered by both Philbrick and Lovell. Before the components of this type of computor are considered in detail it would be well to consider in a general way some of the aspects of precision in computing devices.

PRECISION AND ACCURACY IN COMPUTING DEVICES

In introducing the subject of precision it is well to review first the terminology which is in common use. The word accuracy in general is used to denote how closely a solution conforms to fact. Precision of a solution is an indication of the sharpness of definition or degree of resolution. As an example consider the value of ϵ (the base of natural logarithms), the value 2.718282 is more precise than 2.718. Both values are accurate statements describing the value of the constant ϵ .

In expressing the precision of an analogically computed result, the sharpness of resolution in the components and the measuring devices is the limiting factor. The accuracy of a result depends upon the validity of the analog which is set up. The digital computor is accurate to the degree with which it will perform operations required of it without making mistakes. The UNIVAC with its self checking system would by this definition then exhibit a high degree of accuracy. The precision of the digital machine is determined by the number of digits in the register and the freedom from round-off errors. (Round-off errors are those errors which accumulate in a problem due to rounding off the answer to the number of significant figures being considered.) This then is the limiting case so long as the machine carries the successive approximations to a high enough degree to utilize the full register.

There is considerable difference in the precision possible by the two methods; in the analog instruments the size of the computor, or how well the analogous physical quantity can be measured determines the precision. Thus, an analog computor usually yields results of 3 or 4 figure precision which is good enough for many engineering purposes because the original data may be no better. The digital machine on the other hand, can give nearly any desired precision without an increase in machine size, by exchanging time for more significant figures. In digital machines it is usual to use 10 to 20 significant figures.



FIG. 21 DIRECT ANALOG TYPE COMPUTOR
at California Institute of Technology
Courtesy of California Institute of Technology

Courtesy:

By Korn & Korn

Book Company

Electronic Analog Computors

Published by McGraw Hill

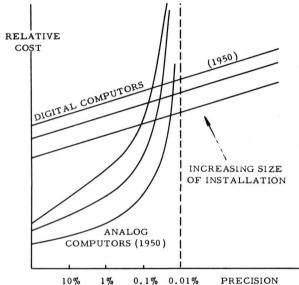
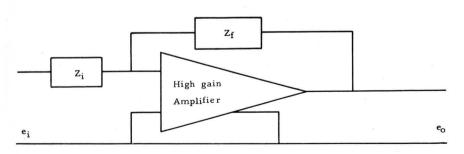


Fig. 22 Relative cost vs precision for digital and analog computors (1950) for different size installations



$$e_o = \frac{z_f}{z_i} e_i$$

SCHEMATIC DIAGRAM OF OPERATIONAL AMPLIFIER COMPUTING ELEMENT

An interesting curve showing relative cost of attaining greater precision in analog and digital machines is shown in Figure 22. As is seen in the curve by Korn and Korn³⁸ the cost of attaining greater precision in a digital machine is essentially linear. Precision in analog computation on the other hand becomes increasingly more expensive. This is understandable since the cost of measuring instruments for physical quantities and the cost of components both rise sharply with the precision required.

How much precision to buy in an analog or a digital type computor is a problem governed mainly by the problem to be solved. For the solution of some structural dynamic problems where the system parameters are no more than 100% precise, and the fabrication is rough, it would hardly be conceivable to purchase digital or analog equipment with a precision of .01%.

ELECTRONIC DIFFERENTIAL ANALYZER

The most recent of the general purpose analog computors is the electronic differential analyzer. This machine, more generally known as the operational analog type, is characterized by such commercially available systems as the BOEING, EASE, GEDA, IDA, GAP/R and REAC. The nucleus of this indirect electrical analog is a high gain d-c amplifier shown in Figure 24 which with appropriate feed-back networks is known as the operational amplifier. Philbrick and Lovell each independently observed that with such an arrangement it would be possible to perform mathematical operations with voltages as the variables. The high gain amplifier by virtue of its position in the circuit assures the accuracy of the mathematical operations in terms of a few well known circuit components. Thus, the precision and general availability in pure form of such components as resistors and capacitors is exploited to perform strict mathematical operations.

While it is not possible because of space limitations to list all of the mathematical operations which it is possible to perform, the basic or simple operations as well as a few of the more common non-linearities would be of interest. Diagrams of the four basic mathematical operations of addition, scale change or proportioning, integrations and differentiation are shown in Figure 25. With these basic operations available, which I shall now designate by boxes with the appropriate symbol as shown in Figure 25, it is possible to solve linear differential equations. As an example consider the equation.

$$\dot{y} = F(t) - a\dot{y} - by$$

This equation states that the summation of terms on the right hand side of the equation should equal the second derivative of the variable y. thus, if these terms are available at the input of the adder (A) as shown in Figure 26 the output will be y. This output can now be integrated twice to yield the appropriate inputs to the adder upon application of the correct scaling factors. The forcing function F (t) may be a perfectly arbitrary function of time so long as it is possible to generate it as an electrical voltage. The usefulness of the computor does not stop at the linear realm. It is also possible to consider non-linear systems such as those involving

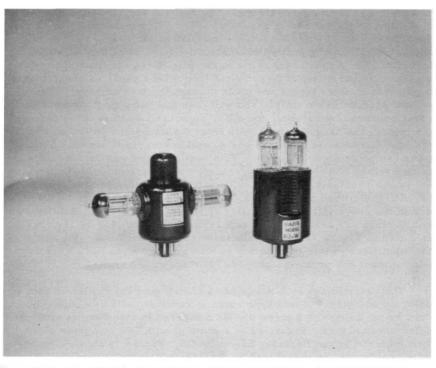


FIG. 24 PLUG-IN TYPE OPERATIONAL AMPLIFIERS

products of two variables, limits, deadzones, or dry friction. These operations are shown schematically in Figure 27. With these concepts available it is now possible to perform an infinite number of combinations of operations to solve linear and non-linear differential equations, algebraic equations, or to perform specific mathematical operations in the process of data reduction.

The analog computor at Pullman, shown in Figure 28, is of the indirect electrical analog type. Although it is commercially manufactured we have modified it to more closely meet our needs. The operational amplifier is of the plug in type shown in Figure 24. The new panels shown in Figure 29 each have eight of these amplifiers mounted in the rear. Each panel has provisions for clamping four of the eight amplifiers when they are used for integration. The direct connections to the amplifiers are made through the triangular configurations of jacks on the front of the panel. Connections through potentiometer inputs are made through the jack adjacent to the individual potentiometer dials. You will note that the upper left hand unit has four potentiometer inputs, the upper right hand unit has three potentiometer inputs, the second row of units from the top has no potentiometer inputs, the third row of units has one potentiometer input, and the bottom row has two potentiometer inputs. This configuration was decided upon after experience had shown us the frequency of occurrence of potentiometer inputs in our problems. The center units are normally used for integrations, differentiations or inverters. A close-up of the panel jacks associated with the upper left hand amplifier is shown in Figure 30. Additional potentiometers are available on a separate panel. The plug-in impedance units shown in Figure 31 are manufactured in our laboratory. We now have eighty amplifiers in this installation as well as provisions for a 100% expansion without any problems of space shortage on the racks.

The illustrations in Figures 32 to 37 will give the reader an idea of some of the equipment of this type which is commercially available. So as to show no partiality the figures will be numbered in alphabetical order to manufacturers names. Figure 32 is a photo of an EASE computor which is manufactured by the Berkeley Scientific Co. Figure 33 shows a photo of a Boeing computor, manufactured by the Boeing Airplane Company. Figure 34 shows the GEDA which is manufactured by the Goodyear Aircraft Co. Figures 35 and 36 show GAP/R computors. The first at Woodward Governor Company is used primarily on "real time" while the second (Figure 36) at Bendix is used principally as a fast time or repetitive computor. Both of these units are manufactured by G. A. Philbrick Research Inc. Finally a REEVES installation is shown in Figure 37. This unit is manufactured by the Reeves Instrument Company.

ACKNOW LEDGEMENTS

The author wishes to express his sincere appreciation to the many people and organizations who have assisted in securing illustrations for this paper. He is especially indebted to Mr. Dick Drake of the Felt and Tarrant Mfg. Company for his encouragement and cooperation during the writing of this paper, and to both Professor V. C. Rideout of the University of Wisconsin and Dr. Rufus Oldenburger of the Woodward Governor Company who edited early copies of this text.

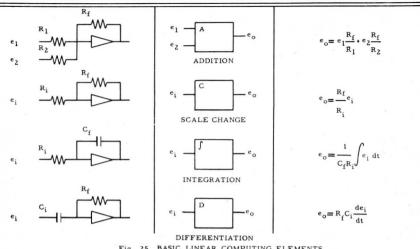


Fig. 25 BASIC LINEAR COMPUTING ELEMENTS

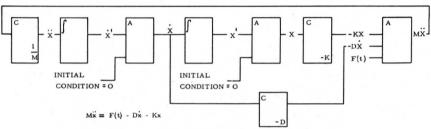


Fig. 26 COMPUTOR BLOCK DIAGRAM FOR THE SOLUTION OF A SINGLE DEGREE OF FREEDOM SYSTEM

| MULTIPLIER | | e ₁ M e ₀ . | | $e_0 = Ke_1e_2$ |
|----------------|----------------------|---|---|--|
| LIMIT | -c +c e ₁ | e _i | for $-C < e_i < C$, for $e_i < -C$, e | e _o ⇒ Ke _i |
| DEAD ZONE | $-c$ $+c$ e_i | e_i Z e_o | for $-C < e_i < C$ for $e_i < -C$, e_i | e _o = 0 |
| DRY FRICTION | +A | e _i — Qperational e _o Amplifier | for $e_i > 0$, for $e_i < 0$, | e _o =+A e _o =-A |
| ABSOLUTE VALUE | e _i | e _i ve _o | | e _o = K e _i |

Fig. 27 NON - LINEAR OPERATIONS

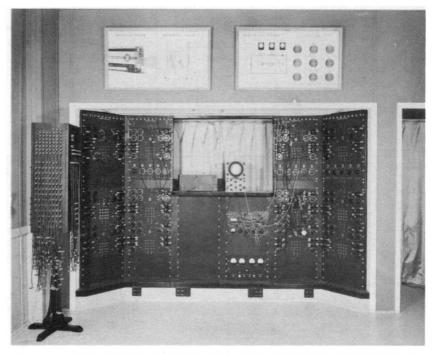


FIG. 28 PULLMAN-STANDARD CAR MANUFACTURING COMPANY'S ANALOG COMPUTOR INSTALLATION

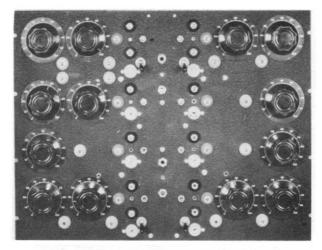
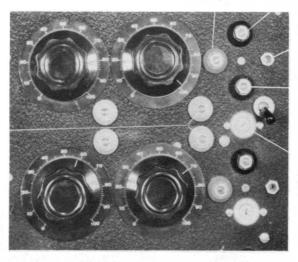


FIG. 29 INDIVIDUAL COMPUTING PANEL - of the Pullman-Standard Analog Computor

Output

Direct Input



Potentiometer Inputs Ground

Balancing Grid

Balancing Potentiometer

FIG. 30 DETAIL OF UPPER LEFT SECTION OF COMPUTING PANEL

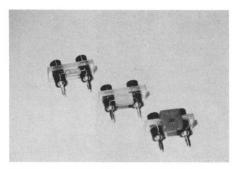


FIG. 31 PLUG-IN IMPEDANCE ELEMENTS - For use with the Pullman-Standard Computor.

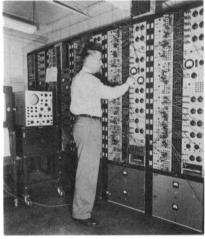


FIG. 33 BOEING ANALOG COMPUTOR
Courtesy of Boeing Airplane Company

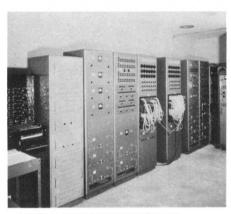


FIG. 32 BERKLEY EASE COMPUTOR
Courtesy of J. B. Rae, Los Angeles

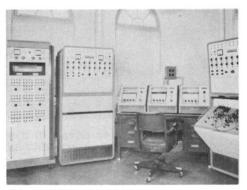


FIG. 34 GOODYEAR GEDA COMPUTOR INSTALLATION Courtesy of Goodyear Aircraft Company, Akron, Ohio

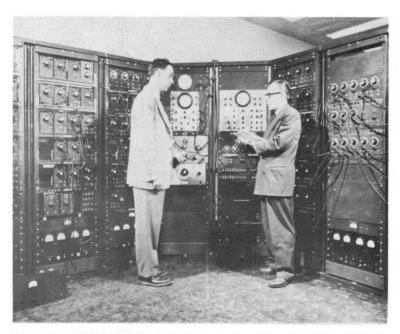


FIG. 35 PHILBRICK ANALOG COMPUTOR INSTALLATION at Woodward Governor Company
Courtesy Woodward Governor Company

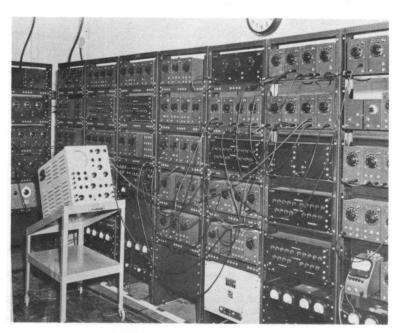


FIG. 36 PHILBRICK COMPUTOR INSTALLATION at Bendix Products Division Courtesy Bendix Aviation Co.

- 1"Maps", Chambers Encyclopedia, Vol. No. 15, 1950, Pages 1-144.
- 2"Maps", Encyclopedia Brittanica, Vol. No. 21, 1949, Page 837.
- 3"Abacus", Chambers Encyclopedia, Vol. No. 1, 1950.
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FIG. 37 REEVES COMPUTOR INSTALLATION
Courtesy Reeves Instrument Co.