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cooling system would appear to make further consideration of present models impracticable.

There have been many statements and many more conjectures as to the light used in television studios. We quote pertinent figures based on our last six-month period of operation. Our average set illumination was in the neighborhood of 1200 foot-candles of incident light. Our average modeling ratio was 2 to 1, while the average light load was slightly more than 50 kw of 110-volt d-c. Our lowest foundation lighting level was 800 foot-candles, a play in which the contrast throughout the set was carried to the upper limit of the Iconoscope. The highest foot-candle reading recorded was slightly less than 2500 foot-candles, a continuity where, obviously, little modeling was attempted.

In our work of the past three years, we feel that we have established a substantial foundation in television studio lighting on which we hope to base an even simpler system. If we appear to have standardized certain assemblies and particular light-sources, this does not mean that our development work has ceased. It continues with renewed vigor as we see our experiments bearing fruit.

SELECTIVE SIDE-BAND TRANSMISSION IN TELEVISION

By

R. D. Kell and G. L. Fredendall

RCA Manufacturing Company, Inc., Camden, N. J.

Summary—*Reproduction of television detail in a selective side band system is treated as a function of the modulation factor and the ratio of vestigial side band to transmitted side band. A comparison with double side-band transmission is included.*

THE fundamental limitation placed upon the amount of detail which may be obtained ultimately in a television picture is the width of the radio-frequency spectrum allotted to a television channel. Six megacycles has been adopted as a standard width by the Radio Manufacturers' Association (R.M.A.). After provision has been made for the sound channel and guard bands and account taken of practical circuit considerations in receiver design there remains a spectrum about 5.25 megacycles wide for the transmission of picture signals.

A problem of first importance is a determination of the position of the television carrier in a spectrum of fixed width and the amplitude and phase characteristics over the spectrum that lead to the transmission of the greatest amount of detail. The following discussion is a mathematical analysis of the problem based upon certain reasonable simplifications.

The amount of picture detail refers to the fidelity of reproduction at the receiver of abrupt changes in intensity of half-tones in the picture at the transmitter. Figure 1 illustrates the typical abrupt changes which may occur in the direction of scanning (horizontal detail). The transmission of similar changes which occur at right angles to the direction of scanning (vertical detail) does not involve the transmission characteristics of the system and thus need not be considered. In (a) and (b) the single abrupt change in intensity is assumed to be isolated to the extent that the corresponding signal is not perceptibly influenced by preceding or following detail. Such detail occurs at the junctions between relatively large areas having different half-tone values.

The pulses in (c) and (d) have a width of the order of a scanning line and correspond to an isolated narrow line perpendicular to the direction of scanning.

Two pulses not necessarily of the same height, but separated by a distance comparable to the width of the pulse are shown in (e) and (f). These correspond to two closely spaced vertical lines in the picture.

Since all types in Figure 1 are fundamental in the building of detail in a television picture, no type can be safely excluded from a study of television transmission.

PREVIOUS STUDIES OF SELECTIVE SIDE-BAND TRANSMISSION

Almost from the beginning of the transmission and reception of television images it was found that a better picture could be obtained with the receiver tuned so that the carrier was located on one side of the selectivity curve.

Poch and Epstein[1] have demonstrated by laboratory measurement the improvement in the reception of detail (e) resulting from moving the carrier to one side of the selectivity curve of a receiver. They gave a mathematical confirmation in the nature of a steady-state analysis of the phase and amplitude characteristics of the video signal corresponding to a low percentage of modulation of the carrier by a single video frequency. In addition to these steady-state conditions, it is important to know the response to the wave forms as shown in Figure 1.*

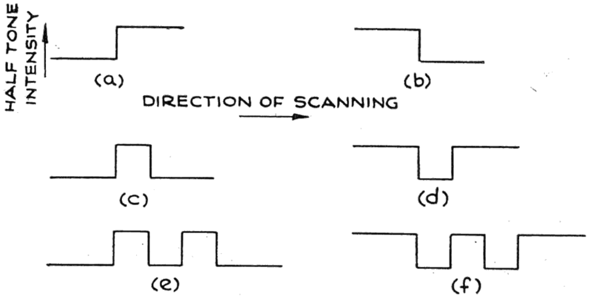

Fig. 1—Television detail.

Goldman[2] has presented a mathematical analysis of the transmission of the details in Figure 1 by a selective side-band system. In his analysis the carrier was varied over a channel of one specific transmission characteristic.

PRELIMINARY CONSIDERATIONS

If the details shown in Figure 1 could be scanned by a pick-up device having an aperture of infinitesimal dimension in the direction of scanning, the video signal generated would have the same waveform as the transitions in half-tone. The effect of a finite symmetrical aperture of a practical scanning device may be obtained by imagining that the signal generated by the infinitesimal aperture is passed through an electrical filter having no phase distortion, but an amplitude distortion characteristic of the particular finite aperture. It has been found possible in practice to compensate electrically for the amplitude distortion at least up to the highest video frequency which conceivably could be accommodated in the proposed television channels. Hence, it shall be assumed in the following treatment that suitable correction has been made and that the transmitting aperture is not a controlling factor in shaping the transmitted signal.

The video amplifiers at the transmitter are regarded as distortionless up to an abrupt cut-off frequency f_o (less than the highest frequency of aperture compensation) and, therefore, become the controlling factor in modifying the waveform of a picture signal before modulation of the radio-frequency carrier. A linear detector responding to the envelope of the intermediate-frequency signal is commonly employed in television receivers and is assumed here.

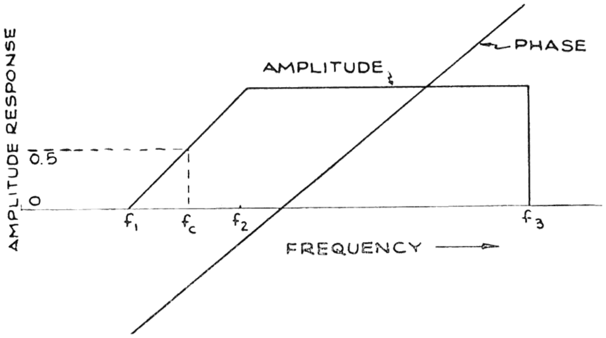

Fig. 2—Idealized transmission characteristic $(f_o - f_c) < (f_1 + f_c)$.

If a receiving aperture of infinitesimal width in the direction of scanning were possible, the variation in light intensity along a scanning line would have the same wave shape as the intermediate-frequency envelope. In practice the receiving aperture is finite, but the amplitude distortion thus introduced may be compensated electrically to a degree that justifies the assumption of an infinitesimal aperture.

With the above suppositions, the envelope of the signal at the input to the detector becomes a direct criterion of the fidelity of transmission.

POSITION OF THE CARRIER ON THE TRANSMISSION CHARACTERISTIC

The overall transmission characteristic of the system properly includes the characteristics of the radio-frequency circuits at the

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* A paper, "Effect of the Quadrature Component in Single Side-Band Transmission," by H. Nyquist and K. W. Pfleger has been published in *B.S.T.J.*, January, 1940, since this manuscript was accepted by the publisher.

transmitter and the radio- and intermediate-frequency circuits of the receiver. Figure 2 shows idealized overall characteristics which (although not physically compatible) presumably could be approximated in an actual system. The amplitude characteristic shows partial suppression of one side band; the phase shift is a linear function of the frequency. The latter assumption is desirable because thereby the best transmission associated with a given amplitude characteristic will be obtained.

If the steady-state amplitude and phase characteristics are derived by determining the envelope of the system when modulated with various video frequencies, one at a time, it is found that the position of the carrier frequency exerts a large effect on the amplitude characteristic. If the carrier is near f_1 in Figure 2, the high-frequency portion is accentuated; if near f_2, the low-frequency portion is accentuated; and if f_c is half way between f_1 and f_2, the frequency response is flat. Harmonics of the modulation frequency are always generated when one side-band is partially suppressed; hence, the conventional frequency response of the system may be misleading unless properly qualified. If the percentage of modulation is sufficiently low, the magnitudes of harmonics are negligible.

In the analysis of selective side-band transmission the carrier f_c shall be fixed at the point of 50 per cent response which gives a flat frequency response. Frequent comparisons will be made between double and selective side-band transmissions.

REPRODUCTION OF A UNIT FUNCTION DETAIL

Figure 3 shows a carrier wave modulated by a signal which is the response of the video frequency amplifiers at the transmitter to a unit function detail. As a consequence of the finite cut-off frequency of the video amplifier, the response (Figure 3b) is not a unit function, but has the same form as that of a low-pass filter to a unit function. The solution for the envelope of the modulated carrier at the output (at the receiver) of the idealized selective side-band system of Figure 2 is derived in Appendix 1 and summarized in Equations (8) and (9).

A quantity $(f_3 - f_c)$ $(t - \tau)$ is the independent variable in which

t is the time,
τ is the time delay of the envelope equal to the slope of the linear phase shift curve,
$(f_3 - f_c)$ is the video band width.

The envelope is found to be a function of two parameters, the ratio $(f_c - f_1)/(f_3 - f_c)$ and a modulation factor m. The modulation factor is determined by the relative amplitudes of the carrier before and after the scanning of an abrupt edge. This is illustrated in Figure 3c. Thus m is equal to unity when one of the carrier levels is zero.

The square root of the sum of the squares of the in-phase component P plus a constant and the quadrature component Q define the shape of the envelope (Equation 9). As the modulation factor approaches zero, the magnitude of the in-phase component becomes large compared with the quadrature component and the envelope approaches the case of double side-band transmission. In double side-band transmission the quadrature component is zero and the in-phase component has the same form as in Equation (8). Thus, the effect of the partial suppression of one side band is to introduce distortion in the form of the quadrature component.

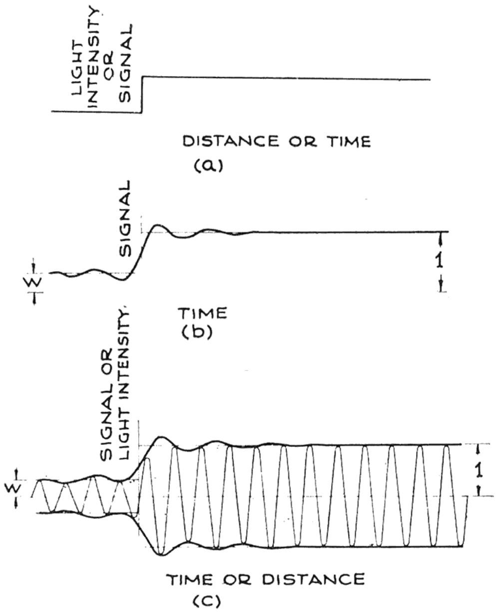


Fig. 3—(a) Unit function detail.
(b) Response of idealized amplifier to (a).
(c) Response of selective side-band system to (b).

$$\text{Modulation factor } m = \frac{1 - W}{1 + W}.$$

In Figure 4 a family of envelopes have been plotted according to Equation (9) in which the partially suppressed side band $(f_c - f_1)$ is the parameter and m is equal to unity. A fixed band width of 5.25 megacycles is used corresponding to the standards of the R.M.A. A more explicit independent variable $(t - \tau)$ is used in place of the generalized form $(f_3 - f_c)$ $(t - \tau)$.

Under the conditions laid down initially, the variation of intensity along the scanning line has the same wave shape as the envelope of the electrical response. Hence, the axis of ordinates may be regarded as the intensity and the axis of abscissas as the distance along the scanning line.

A distance equal to one scanning line pitch corresponds to 0.12 microsecond (R.M.A. Standards).

Several observations may be made when $m = 1$.

(1) The steepness of rise for different values of $(f_c - f_1)$ do not differ significantly in the interval -0.15 to 0 microseconds, the range of greatest variation in the response.

(2) The amount of "transient" overswing of each envelope increases as $(f_c - f_1)$ decreases. Under the R.M.A. standard that white correspond to zero carrier, the overswing and damped oscillation would appear as striations of alternate light and dark bands superimposed in the gray region of the white to gray transition. The first striation is wider than a picture element and would, therefore, be visible at the correct viewing distance from the screen. Succeeding striations are of the order of a picture element and, therefore, may not be distinguishable.

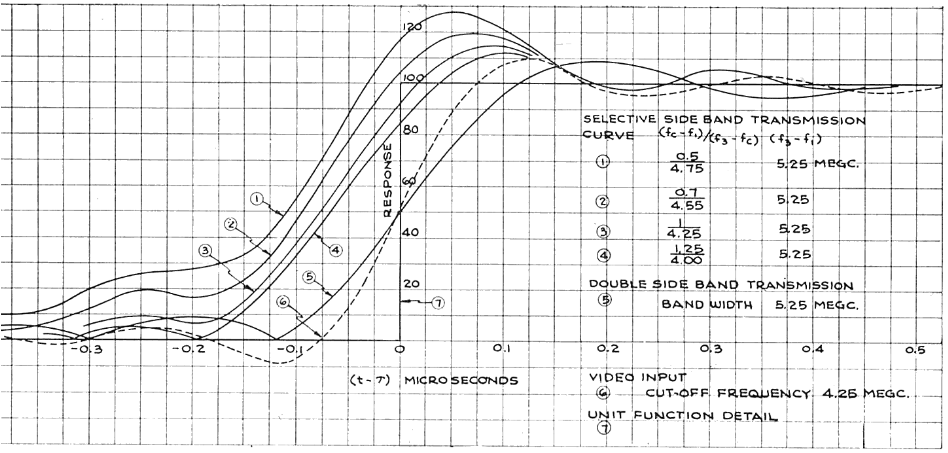


Fig. 4—Transmission of unit function detail.

(3) The principle rise that largely identifies the location of the transition in the received picture is preceded by an anticipatory step which is more pronounced as $(f_c - f_1)$ is decreased. This step gives the visual impression of a blurred transition. In this respect systems represented by curves (3) or (4) are definitely superior.

(4) The fidelity with which a unit function is transmitted through a system characterized by a particular value of $(f_c - f_1)$ may be judged by comparing the envelope of the response with the corresponding video signal which supplies the modulation. It is recalled that the video signal at the transmitter is the output of an idealized amplifier having an abrupt cut-off frequency equal to $f_o = (f_3 - f_c)$. Thus, envelope (3) drawn for $(f_c - f_1) = 1$ megacycle must be compared with curve (6), the video response to a unit function detail of an idealized amplifier for which $(f_3 - f_c) = 4.25$ megacycles.

The ratio of the average slope (in the region of principal rise) of curve (3) to that of curve (6) is about 1.6. Thus, an abrupt transition between half-tones appears less abrupt when received in the selective side-band system (curve 3) than when applied as a modulating signal at the transmitter (curve 6).

(5) Curve (5), the envelope of the response of a double side-band system 5.25 megacycles wide, has an average steepness comparable with those of the envelopes (3) and (4) for selective side-band transmission. This means that for a fixed band width there is no superiority of a selective side-band system over a double side-band system for the transmission of a unit function detail at percentages of modulation near 100 (m near unity).

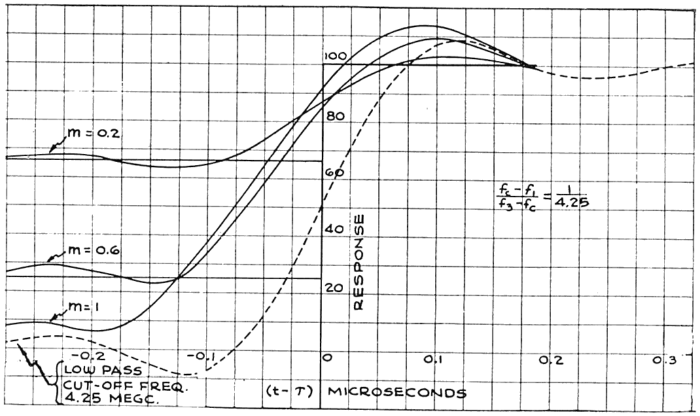


Fig. 5—Transmission of unit function detail as function of modulation factor m.

In a video signal as in an audio signal the average percentage modulation is low. Many of the abrupt transitions in a television subject take place between two half-tones neither of which is white, that is, the value of the modulation factor m is not unity. Figure 5 is a family of envelopes drawn for $m = 0.2$, 0.6, and 1. $(f_c - f_1)$ is taken equal to 1 megacycle and the band width is 5.25 megacycles as in Figure 4. It is observed that the envelopes properly scaled (Figure 6) approach the video signal as m approaches zero. That is, the received signal resembles more and more the modulating video signal (cut-off frequency $= (f_3 - f_c)$ and in this sense becomes distortionless in the limit. This signifies that the fidelity of the idealized selective side-band system approaches that of a double side-band system having a band width of 8.5 megacycles for small percentages of modulation and that of a double side-band system 5.25 megacycles wide for percentages of modulation near 100.

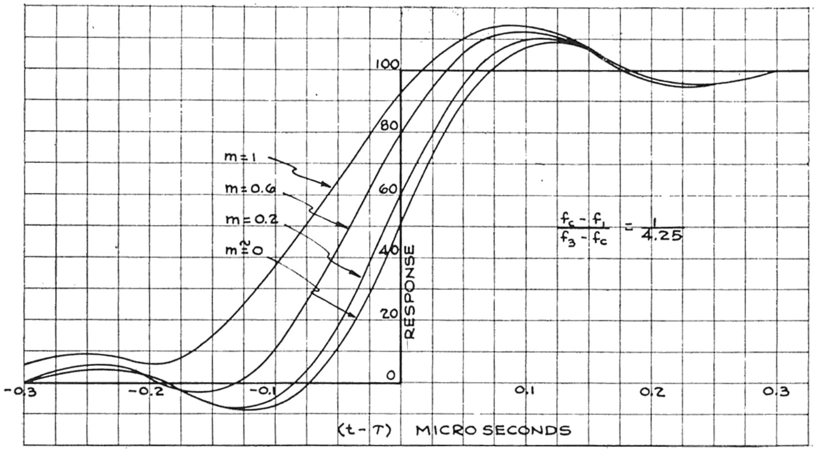


Fig. 6—Transmission of unit function detail as function of modulation factor *m*.

REPRODUCTION OF NARROW LINE DETAIL

A narrow line perpendicular to the direction of scanning is ideally represented by the square pulse in Figure 7(1). When the signals generated by the scanning device are limited in the amplifiers to a band width of 4.25 megacycles, the video waveforms for a pulse 0.15 microsecond long are shown in Figure 7, curve (2). This pulse corresponds to a line having a width approximately equal to a picture element in the present television system. Except for the negative loops which should be reflected in the time axis curve (2) is also the envelope of a carrier modulated by the pulse with a modulation factor equal to one and transmitted double side band with a band width of 8.5 megacycles. Curve (3) indicates that the maximum amplitude of the response of a selective side-band system 5.25 megacycles wide is the same as that of a double side-band system 8.5 megacycles wide. Curve (4) is the response when a band width of 5.25 megacycles is used for double side-band transmission. Comparing the two modes of transmission on the same band width it is observed that a narrow line is reproduced at only 70 per cent of its proper intensity in the double side-band case.

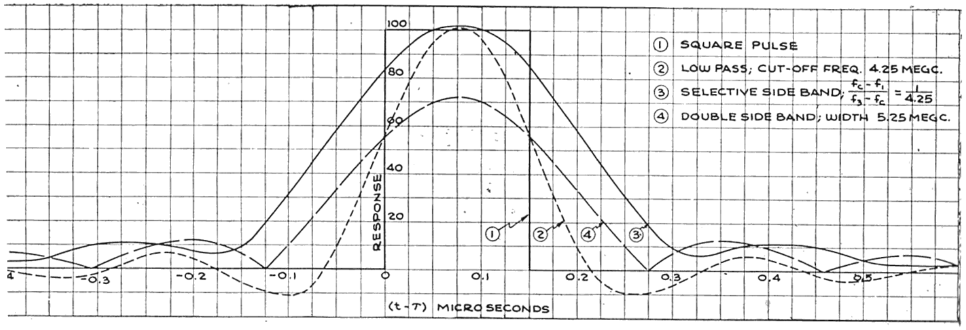


Fig. 7—Transmission of narrow line detail.

The apparent width is also a significant characteristic of lines of the order of a picture element wide. There is an apparent elongation of the pulse after transmission through a selective side-band system. As the modulation factor is reduced there is less and less distortion, and in the limit the transmission is the same as through a double side-band system 8.5 megacycles wide.

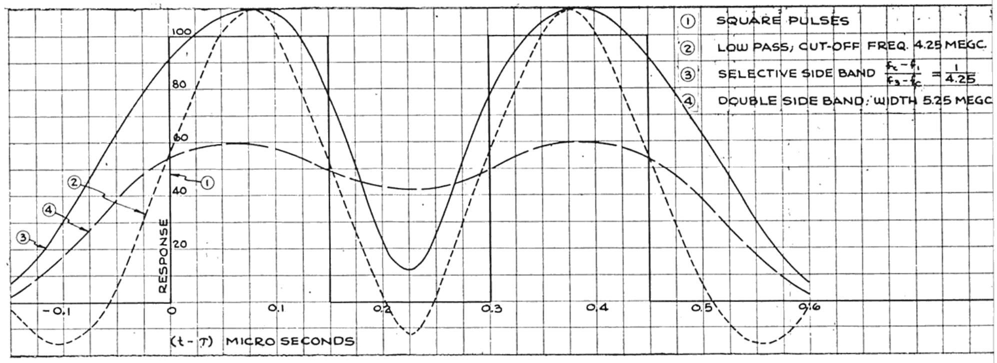


Fig. 8—Transmission of two closely spaced narrow lines.

REPRODUCTION OF CLOSELY SPACED NARROW LINES

The term "resolution" is applied most frequently to the property of a system to distinguish between closely spaced narrow lines as represented in Figure 1e. Mathematical expressions have been developed in Appendix 3 that afford a revealing comparison of the fidelity of selective and double side-band systems for the reproduction of such fine line detail. A series of envelopes corresponding to two square pulses 0.15 microsecond long and separated by an interval of the same length are shown in Figure 8. It is observed that the video signal, curve (2), provides resolution of the two lines by sinking to a sustained low value between lines. Curve (3) represents the response of a selective side-band system and does contain an interval of low signal value, but curve (4) for double side-band transmission over a band of equal width does not indicate appreciable resolution. This result is certainly not predicted in Curve (3), Figure 7, which illustrated the case of a single narrow line. The sloping edge of the reproduced single pulse itself exceeded the separation of pulses in Figure 8. An examination of the mathematical expression for the envelope (Equation 11) affords the explanation of the apparent contradiction. The quadrature or distorting components due to the first and second pulses partially cancel near the center of the separating interval and exactly cancel at the center.

ECONOMY IN BAND WIDTH

The economy in band width obtained by means of partial suppression of one side band depends therefore upon the range of the modulation factor. Thus, curves may be drawn for each type of television detail showing the relation between the modulation factor and the band width required for equal fidelity of reproduction of horizontal and vertical detail. This would involve a method of measuring the sharpness of transition between halftones in the unit function detail, the width of a narrow line, and the resolution of two narrow lines. The subject of the measures of vertical and horizontal resolutions in a television picture is variously treated by writers and it is not the purpose of this discussion to enter into the merits of different methods. The permissible range of the modulation factor in a television picture will represent a compromise between fidelity, band width, and intensity of the transmitted signal.

CONCLUDING REMARKS

The intent of this analysis has been to demonstrate the characteristic differences of double and selective side-band transmissions of television detail and in particular to find a favorable transmission characteristic for the latter. The analysis was developed on the hypothesis of a linear phase characteristic throughout the pass band in order to obtain the optimum envelope associated with a given amplitude characteristic. Thus, the envelopes derived here cannot be duplicated point for point in a physically possible system in which some phase distortion always resides, but the broad aspects of the idealized treatment may be realized in practice when careful phase compensation has been provided. The preceding work may be summarized as follows:

(1) If the modulation factor is near 1, a unit function detail is transmitted most faithfully in a selective side-band system when the ratio of the partially suppressed side band to the completely transmitted side band $(f_c - f_1)/(f_3 - f_c)$ has a value lying between 1/4.25 and 1.25/4. Comparable fidelity is obtained in a double side-band system of equal band width that requires a video band width appreciably less than in the selective side-band example. As the modulation factor becomes less than 1, the fidelity of the selective side-band system for the transmission of the unit function increases, whereas that for double side-band transmission (equal band width) does not increase. In the limit as the modulation factor approaches zero, the sharpness of reproduction in the selective side-band system is about 1.6 times greater.

(2) When the modulation factor is equal to 1 the width of the input video signal corresponding to a single narrow line is increased about equally after transmission through either system operated over equal band widths, but there is a reduction in amplitude of the envelope in a double side-band system. The extension in width approaches zero as the factor is made progressively smaller in the selective side-band system, but there is no change in the other system.

(3) Two narrow lines are resolved more completely by selective side-band transmission for any value of the modulation factor than by double side-band transmission over an equal band width. As the factor becomes less than 1, the remarks above also apply for the resolution of narrow lines.

APPENDIX

I. RESPONSE OF A SELECTIVE SIDE-BAND SYSTEM TO A UNIT FUNCTION DETAIL

Figure 3b may be regarded as the limit of a square wave $E(t)$ as the fundamental frequency approaches zero and the upper limit of the frequency spectrum is held constant.

$$E(t) = \frac{1}{1+m}\left[1 + \frac{4m}{\pi}\sum_{1}^{N}\frac{\sin(2n-1)}{(2n-1)}\omega t\right] \tag{1}$$

$$(2N-1)\omega = \omega_o$$

A sine-wave carrier modulated by $E(t)$ has the form

$$e_1(t) = \frac{1}{1+m}\left\{\sin\omega_c t + \frac{2m}{\pi}\sum_{1}^{N}\cos\left[\omega_c - (2n-1)\omega\right]t\right.$$

$$\left.\frac{-2m}{\pi}\sum_{1}^{N}\cos\left[\omega_c + (2n-1)\omega\right]t\right\}.$$

If $e_1(t)$ is impressed on a linear system that alters the amplitude and phase, there results

$$e(t) = \frac{1}{1+m}\left\{A_c\sin(\omega_c t + \theta_c) + \frac{2m}{\pi}\sum_{1}^{N}\frac{A_{(2n-1)}}{2n-1}\cos\left\{\left[\omega_c - (2n-1)\omega\right]t + \theta(2n-1)\right\}\right. \tag{2}$$

$$\frac{-2m}{\pi}\sum_{1}^{N}\frac{B_{(2n-1)}}{2n-1}\cos\left\{[\omega_c+(2n-1)\omega]t+\beta(2n-1)\right\}.$$

If the phase shift is linear then

$$\theta_{(2n-1)}=\tau[\omega_c-(2n-1)\omega]+b$$

$$\beta_{(2n-1)}=-\tau[\omega_c+(2n-1)\omega]+b$$

$$T=(t-\tau).$$

(2) becomes

$$\begin{aligned}
e(t) &= \frac{1}{1+m}A_c\sin(\omega_cT+b)\\
&+\frac{2m}{\pi}\cos(\omega_cT+b)\sum_{1}^{N}\frac{A_{(2n-1)}}{2n-1}\cos(2n-1)\omega T\\
&+\frac{2m}{\pi}\sin(\omega_cT+b)\sum_{1}^{N}\frac{A_{(2n-1)}}{2n-1}\sin(2n-1)\omega T\\
&-\frac{2m}{\pi}\cos(\omega_cT+b)\sum_{1}^{N}\frac{B_{(2n-1)}}{2n-1}\cos(2n-1)\omega T\\
&+\frac{2m}{\pi}\sin(\omega_cT+b)\sum_{1}^{N}\frac{B_{(2n-1)}}{2n-1}\sin(2n-1)\omega T].
\end{aligned}\qquad(3)$$

(3) has the form

$$\left(P+\frac{A_c}{1+m}\right)\sin(\omega_cT+b)+Q\cos(\omega_cT+b)=\sqrt{\left(P+\frac{A_c}{1+m}\right)^2+Q^2}\;\cos[\omega_cT+b+\varepsilon]\qquad(4)$$

where

$$P=\frac{1}{1+m}\;\frac{2m}{\pi}\sum_{1}^{N}\left\{\frac{A_{(2n-1)}}{2n-1}+\frac{B_{(2n-1)}}{2n-1}\right\}\sin(2n-1)\omega T\qquad(5)$$

$$Q=\frac{1}{1+m}\;\frac{2m}{\pi}\sum_{1}^{N}\left\{\frac{A_{(2n-1)}}{2n-1}-\frac{B_{(2n-1)}}{2n-1}\right\}\cos(2n-1)\omega T.$$

The envelope of the modulated carrier is the coefficient

$$\sqrt{\left(P+\frac{A_c}{1+m}\right)^2+Q^2}.$$

$A_{(2n-1)}$ and $B_{(2n-1)}$ may be assigned values in accordance with the amplitude characteristic of Figure 2.

$$\frac{A_{(2n-1)}}{2n-1}+\frac{B_{(2n-1)}}{2n-1}=\frac{1}{2n-1}\quad\text{over the characteristic.}$$

P becomes

$$\frac{1}{1+m}\;\frac{2m}{\pi}\sum_{1}^{N}\frac{\sin(2n-1)\omega T}{2n-1}.$$

$$\frac{A_{(2n-1)}}{2n-1}-\frac{B_{(2n-1)}}{2n-1}=-\frac{\omega}{\omega_c-\omega_1}\quad\text{on the sloping part of the characteristic and}$$

$$\frac{A_{(2n-1)}}{2n-1}-\frac{B_{(2n-1)}}{2n-1}=-\frac{1}{2n-1}\quad\text{on the straight part of the characteristic.}$$

Q becomes

$$\frac{1}{1+m}\left[-\frac{2m}{\pi}\;\frac{\omega}{(\omega_c-\omega_1)}\sum_{1}^{p}\cos(2n-1)\omega T-\frac{2m}{\pi}\sum_{p+1}^{N}\frac{1}{2n-1}\cos(2n-1)\omega T\right]\qquad(6)$$

where $(2p-1)\omega=(\omega_c-\omega_1)$; $(2N-1)\omega=\omega_o=(\omega_3-\omega_c)$.

The first sum in (6) may be simplified by using the following proposition[4]

$$\sum_{1}^{\frac{K+2}{2}}\cos(2n-1)\theta=\frac{\frac{1}{2}\sin(K+2)\theta}{\sin\theta}.$$

There results

$$\sum_{1}^{p}\cos(2n-1)\omega T=\frac{\frac{1}{2}\sin 2p\omega T}{\sin\omega T}=\frac{\frac{1}{2}\sin(\omega_c-\omega_1+\omega)T}{\sin\omega T}.$$

If $\omega \rightarrow o$ there results

$$P = \frac{m}{1+m} \frac{1}{\pi} \int_{o}^{(\omega_3 - \omega_c)T} \frac{\sin x}{x} dx$$

$$Q = \frac{m}{1+m} \frac{1}{\pi} \left[- \frac{\sin (\omega_c - \omega_1) T}{(\omega_c - \omega_1) T} - \int_{(\omega_c - \omega_1)T}^{(\omega_3 - \omega_c)T} \frac{\cos x}{x} dx \right. \tag{7}$$

$$= \frac{m}{1+m} \frac{1}{\pi} \left[- \frac{\sin (\omega_c - \omega_1) T}{\omega_c - \omega_1} - \int_{(\omega_c - \omega_1)T}^{\infty} \frac{\cos x}{x} dx + \int_{(\omega_3 - \omega_c)T}^{\infty} \frac{\cos x}{x} dx. \right.$$

If a change of independent variable is made in (7)

$$(\omega_3 - \omega_c)\ T = \eta$$

and if

$$\frac{\omega_c - \omega_1}{\omega_3 - \omega_c} = \delta$$

more general forms for P and Q are (8)

$$P = \frac{m}{1+m} \frac{1}{\pi} \int_{o}^{\eta} \frac{\sin x}{x} dx$$

$$Q = \frac{m}{1+m} \frac{1}{\pi} \left[- \frac{\sin \eta\delta}{\eta\delta} - \int_{\delta\eta}^{\infty} \frac{\cos x}{x} dx + \int_{\eta}^{\infty} \frac{\cos x}{x} dx \right].$$

δ and m are parameters.

These integrals have been tabulated extensively[5].

According to (4) the envelope is the coefficient

$$\sqrt{\left\{ P + \frac{1}{2} (1+m) \right\}^2 + Q^2}. \tag{9}$$

PART 2. RESPONSE OF A SELECTIVE SIDE-BAND SYSTEM TO A SQUARE PULSE

The equation of a square pulse T_1 seconds long is obtained by adding a unit function having an amplitude $\left(- \frac{2m}{1+m} \right)$ and delayed T_1 seconds to $E(t)$, Equation (1). The solution for the corresponding envelope follows in a manner similar to the development in Part 1. The result is

$$\text{envelope} = \sqrt{\rho^2 + \partial^2}$$

$$\text{where } \rho = \frac{1}{2(1+m)} - \frac{m}{2(1+m)} + P(T) - P(T - T_1) \tag{10}$$

$$\partial = Q(T) - Q(T - T_1).$$

The P and Q functions are defined by (7).

PART 3. RESPONSE OF A SELECTIVE SIDE-BAND SYSTEM TO TWO SQUARE PULSES

Two pulses illustrated in Figure 1c are formed by adding unit functions of the following descriptions to $E(t)$:

$$\text{amplitude } \frac{-2m}{1+m}\text{; delayed } T_1 \text{ seconds}$$

$$\text{amplitude } \frac{2m}{1+m}\text{; delayed } T_2 \text{ seconds}$$

$$\text{amplitude } \frac{-2m}{1+m}\text{; delayed } T_3 \text{ seconds.}$$

The envelope of the response is

$$\sqrt{\rho^2 + \partial^2}$$

where

$$\rho = \frac{1}{2(1+m)} - \frac{m}{2(1+m)} + P(T) - P(T - T_1) + P(T - T_2) - P(T - T_3) \tag{11}$$

$$\partial = Q(T) - Q(T - T_1) + Q(T - T_2) - Q(T - T_3).$$

Block-shaped signals of any description may be expressed by suitably combining functions of the unit function type. The envelopes will be given by P and Q functions defined in (7).

PART 4. RESPONSE OF LOW-PASS SYSTEMS TO TELEVISION DETAIL

a. Unit Function (Figure 1a)

$$e(t) = \frac{1}{2} + \frac{1}{\pi} \int_{o}^{2\pi f_o T} \frac{\sin x}{x} dx. \tag{12}$$

b. Square Pulse (Figure 1c)

$$e(t) = \frac{1}{\pi}\left[\int_{o}^{2\pi f_o T} \frac{\sin x}{x}\,dx - \int_{o}^{2\pi f_o (T - T_1)} \frac{\sin x}{x}\,dx\right]. \quad (13)$$

c. Two Square Pulses (Figure 1e)

$$e(t) = \frac{1}{\pi}\left[\int_{o}^{2\pi f_o T} \frac{\sin x}{x}\,dx - \int_{o}^{2\pi f_o (T - T_1)} \frac{\sin x}{x}\,dx + \right. \quad (14)$$

$$\left. \int_{o}^{2\pi f_o (T - T_2)} \frac{\sin x}{x}\,dx - \int_{o}^{2\pi f_o (T - T_3)} \frac{\sin x}{x}\,dx\right]$$

V. RESPONSE OF DOUBLE SIDE-BAND SYSTEMS TO TELEVISION DETAIL

Same as in Part 4 if $f_o = \dfrac{\text{band width}}{2}$.

LIST OF REFERENCES

[1] Poch and Epstein, "Partial Suppression of One Side Band in Television Reception," RCA REVIEW, Vol. I, p. 19, 1937.

[2] Goldman, "Television Detail and Selective Side-band Transmission," I. R. E. Fall Convention (1938), Rochester, New York.

[3] Poch and Epstein, *loc. cit.*

[4] Chrystal's Algebra II, p. 273.

[5] Jahnke-Emde, "Tables of Functions," second revised edition, p. 78. B. G. Teubner, Leipsig and Berlin, (1933).

FLUCTUATIONS IN SPACE-CHARGE-LIMITED CURRENTS AT MODERATELY HIGH FREQUENCIES

BY

B. J. THOMPSON, D. O. NORTH AND W. A. HARRIS

RCA Manufacturing Company, Inc., Harrison, N. J.

PART II—DIODES AND NEGATIVE-GRID TRIODES

BY DWIGHT O. NORTH

Summary—*A quantitative theory of shot effect in the parallel-plane diode is formulated for any degree of space charge, and for frequencies such that transit-time effects are of no concern. Beginning with the steady-state description of a diode showing a Maxwell-Boltzmann distribution of emission velocities, the theory is founded upon a determination of the new steady state which results from the injection of a small additional emission comprised of electrons of a specified velocity.*

The fluctuations in a diode current I are expressed by

$$\overline{i^2} = \Gamma^2 \cdot 2eI\Delta f.$$

For a temperature-limited diode it has long been known that $\Gamma^2 = 1$. *For a diode with anode potential sufficiently negative (retarding field),* $\Gamma^2 = 1$ *also. For the usual space-charged-limited case, i.e., any instance in which there is a virtual cathode,* Γ^2 *is less than unity, but in this paper* Γ^2 *is computed by numerical integration for only those instances in which the anode current is a small fraction of the emission.*

In this last case the shot-effect formula is also written, phenomenologically, to correspond with Nyquist's well-known expression for thermal agitation in a passive network of conductance g, thus:

$$\overline{i^2} = \theta \cdot 4kTg\Delta f.$$

Here T is absolute cathode temperature and g is diode conductance. The dimensionless factor θ *is found to be virtually a constant* $\left(\theta \approx \dfrac{2}{3}\right)$ *for the whole range of normal anode potentials with asymptotic value,*

$$\theta \sim 3\left(1 - \frac{\pi}{4}\right)$$

in the limit of high anode potential. That is, the mean-square noise generated by emission fluctuations is roughly numerically equal to two-thirds of the noise of thermal agitation generated by a resistance of magnitude equal to the a-c resistance of the diode and possessing a temperature equal to the cathode temperature.

The theory is extended to cover shot effect in the anode circuit of a triode with negative grid. Unless the amplification factor μ *is very low, the same formulas apply, except that g is now interpreted as the con-*

ERRATA

The Service Range of Frequency Modulation

By

Murray G. Crosby

RCA Review Vol. IV, No. 3, January 1940

Sixth line from bottom of Page 350, change the constant "2.21" "3550".

Delete the last sentence of the third paragraph reading: "However, r this type of noise, etc.," on Page 369.

Replace Figures 7, 8, 9, and 10 with the following corrected agrams.

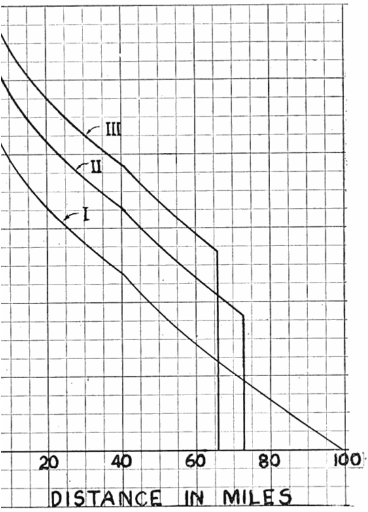


Fig. 7

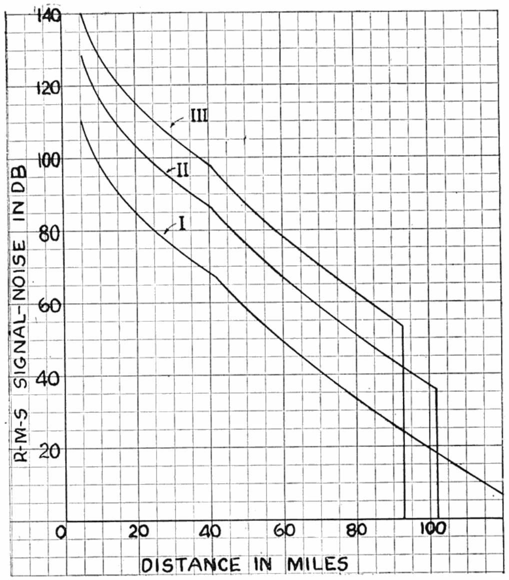
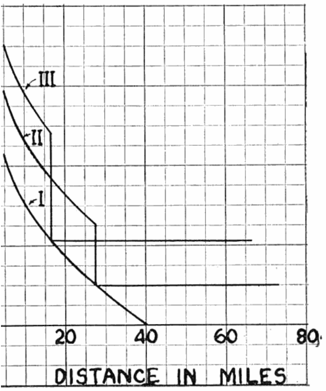
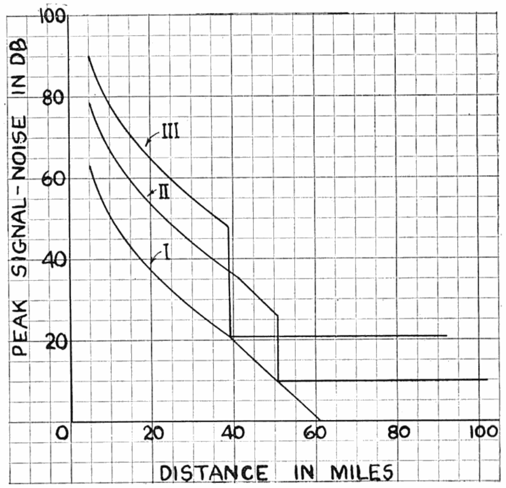


Fig. 8





OUR CONTRIBUTORS

Murray G. Crosby joined the branch of the Radio Corporation of America which is now R.C.A. Communications, Inc., in 1925. He was engaged in the operating and design departments of that branch until 1926, when he took a leave of absence and returned to the University of Wisconsin for one semester and received his degree of B.S. in Electrical Engineering. Since that time he has remained in the research and development division of R.C.A. Communications, Inc. Mr. Crosby is a member of the Institute of Radio Engineers and a fellow of the Radio Club of America.

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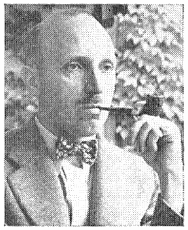
Albert W. Protzman has been connected with radio since 1922. He became Field Supervisor for station WEAF in 1924, and later Field Supervisor for NBC. Mr. Protzman served as Assistant Sound Director for the Fox Film Corporation in Hollywood from 1929 to 1936, when he joined the Television staff of the National Broadcasting Company, where he since has been engaged as a Television Technical Director.



Walter van B. Roberts is a graduate of Princeton University. Before the World War he was connected with the Western Electric Company. He became Head of the Department of Radio and Signalling in the School of Military Aviation in June, 1917, and from March, 1918 to the close of the war he was technical officer of Sound Ranging Section Number 1, on the American front. He later taught in Princeton University from 1919 to 1924. In 1924 he joined the RCA Technical and Test Department and in 1927 transferred to the Patent Department of RCA. Dr. Roberts is a fellow of the Institute of Radio Engineers.

Allen H. Schooley was graduated with the degree of B.S. in Electrical Engineering from Iowa State College in 1931. In 1932 he received his M.S. degree from Purdue Unviersity. Between 1932 and 1936 he did radio servicing, was a computor for the United States Coast and Geodetic Survey, and spent a year at the State University of Iowa doing graduate work in engineering and physics. Mr. Schooley joined the RCA Radiotron Division in 1936, and is now an engineer in the Research and Engineering Department of the RCA Manufacturing Company at Harrison, N. J. He is an associate member of I.R.E., and a member of Sigma Xi.

Newell R. Smith received his degree of B.S. in Electrical Engineering from Ohio University in 1926. Following graduation he served a few months with the Cleveland Illuminating Company prior to joining the Vacuum Tube Standardizing Department of the General Electric Company at Nela Park in Cleveland, Ohio. In 1930 this activity was transferred to the Radiotron Division of the RCA at Harrison, N. J. In 1933 he was made assistant in charge of Standardization in which capacity he remained until 1937 when he became associated with the receiving tube design activity.

Frank E. Spaulding, Jr., attended Yale Engineering School and received his B.S. degree in electrical engineering from the Worcester Polytechnic Institute in 1933. In 1930 he was employed by the Brooklyn Edison Company, and from 1933 to 1935 was with the Underwood Elliot Fisher Company. From 1935 to 1938 he was in the radio receiver engineering department of the General Electric Company at Bridgeport, Conn. Since 1938, Mr. Spaulding has been associated with the Radiomarine Corporation of America, where he is engaged in the development of marine radio equipment. He is an associate member of I.R.E.

Gilbert S. Wickizer, a native of Pennsylvania, received the B.S. degree in electrical engineering from the Pennsylvania State College in 1926. The same year he joined the Radio Corporation of America, Operating Division, and in 1927 transferred to the Receiver Research and Advanced Development Section of R.C.A. Communications, Inc. Mr. Wickizer is a member of Eta Kappa Nu and an associate member of the Institute of Radio Engineers.