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# THE LIMITS OF INHERENT FREQUENCY STABILITY 

## By

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GREAT improvements in oscillator stability have been made in recent years by careful attention to the mechanical design and layout of component parts and also by the development of a number of compensation arrangements. An example of the latter is the compensation for a change in plate voltage by a suitable change in screen voltage. By the proper use of such refinements and by other precautions, frequency variations due to variations of input and output impedances of the tube may be much reduced. However, it seems obvious that if the circuit could be made more stable in the first place, the addition of these schemes would bring about still better final results.

The many causes of frequency variations may be divided into three groups: First, changes in the constants of the frequency determining circuit itself; second, changes introduced by the loading on the circuit; and third, changes in the effective input and output impedances of the oscillator tube which are reflected into the circuit by the necessary coupling of the tube to the circuit. In what follows, only the third group will be considered, and the term "inherent stability" is used to refer to the extent to which the frequency is independent of small changes in the effective tube impedances. The object of this investigation is therefore to determine just how far one can go in reducing the effect of given capacitance changes in a tube on the frequency of any ordinary oscillator circuit.

Figure 1 shows a representative simple feed-back oscillator in which the small capacitances $C_{g}$ and $C_{p}$, represent the maximum variations that may be expected in the input and output circuits of the tube. It is of course possible that these variations may sometimes take place in opposite senses so as to tend to compensate for cach other. However, in order to deal with the worst case possible they will be assumed to take place in the same sense and at the same time. In this case the resulting frequency shift, measured in cycles per
second, may readily be shown to be given approximately by the expression

$$
\begin{equation*}
\frac{f\left(\omega M_{g}\right){ }^{2} C_{!/}}{2 L}+\frac{f\left(\omega M_{\rho}\right)^{2} C_{P}}{2 L} \tag{1}
\end{equation*}
$$

in which $f$ is the oscillator frequency in cycles per second, os is $2 \pi f$, the inductance $L$ and the mutual inductances $M_{g}$ and $M_{i}$, are measured in henrys and the capacitances are in farads.

If now the mutuals are reduced to the point where the system just barely oscillates and if, furthermore, their ratio is adjusted to give the least possible frequency shift when $C_{g}$ and $C_{l}$ disappear or reappear, then the expression

$$
\begin{equation*}
f\left(\frac{r}{L}\right)\binom{\sqrt{C_{g} C_{P}}}{--\cdots} \tag{2}
\end{equation*}
$$

gives the smallest frequency shift that can be obtained in the presence of the capacitance variations $C_{g}$ and $C_{P}$. In this expression $r$ is in ohms and $g$ is the transconductance of the tube in mhos. The derivation of the expression will be given in the appendix.


Fig. 1
From expression 2 it can be seen that the stability is limited by three independent factors. First of all, the minimum possible frequency shift in cycles is seen to be proportional to the frequency of operation, which is natural enough and is merely a way of saying that the percentage frequency shift is independent of frequency. Secondly, this shift is proportional to the ratio $r / L$ which means that a good coil is desirable, a conclusion that again is less than startling. Finally, the shift is proportional to the ratio of the geometric mean of the tube capacitance variations to the transconductance. This result is a little less obvious and might lead to the choice of a tube not ordinarily considered particularly well suited to oscillator use. For example, a certain tube may have a rather large variation of input
capacitance, say 1.0 mmf . Nevertheless, if its output capacitance is constant to within 0.01 mmf the geometric mean variation is only 0.1 mmf and the tube is preferable, other things being equal, to one which has only say 0.2 mmf variation at most, but has this much variation of both its input and output capacitances.

In the foregoing it was assumed that the circuit was just barely oscillating, and the looser the couplings can be made the greater the stability up to the limit given by expression 2. In practice, of course, the couplings would be made somewhat closer to aliow a factor of safety in starting the oscillation, and also, to obtain a sufficiently strong oscillation to be of some use. However, for any given factor of safety, expression 2 will be proportional to the actual frequency variation so that conclusions drawn from it will still be valid.

## Harmonic Operation

When an oscillator is used to obtain excitation in several frequency bands, it is common practice to run it at the frequency of the lowest band, or even a submultiple thereof, and to obtain excitation for the other bands by frequency multiplication. Let us see what conclusions can be drawn from expression 2 regarding this mode of operation. To be specific, suppose the fundamental frequency is in the range from about 830 to 1.020 kilocycles, say 900 kc . (This range is very easily calibrated by beating with broadcast stations and also by beating the sixth harmonic of 833.333 kc and the fifth harmonic of 1000 kc with the 5 -megacycle transmissions of WWV). The number of cycles shift is given by expression 2, and the number of cycle shift of the 28.8 megacycle harmonic is thirty-two times as great since the latter frequency is the thirty-second harmonic of the oscillator. Thus, the formula for the shift in the ten-meter band would be $28,800,000$ $\left(\frac{r}{L}\right)\left(\frac{\sqrt{C_{j} C_{P}^{-}}}{g}\right)$. But this is the same formula that would be used if the oscillator were running at 28.8 megacycles as its funda-

$$
\sqrt{C_{y} C_{P}}
$$

mental except that the ratio $r / L$ and the ratio $-\quad-\cdots$ were evaluated
at 900 kilocycles in the one case and at 28.8 megacycles in the other. Hence, it is seen that expression 2 may be generalized to take care of harmonic operation as follows:

$$
f \text { radiated }\left(\begin{array}{cc}
\gamma & \sqrt{V_{C}} \bar{C}_{P}  \tag{3}\\
\bar{L}
\end{array}\right) \text { fundamental }
$$

where the subscripts indicate that the frequency is taken as the radi-
ated frequency while the rest of the expression is evaluated at the fundamental oscillation frequency.

The interesting thing about expression 3 is that it indicates that for any given radiated frequency the actual number of cycles shift caused by tube variations can be reduced in theory by using a low enough fundamental oscillation frequency. This is partly because the ratio of tube capacitance changes to transconductance is somewhat lower at the lower frequencies, but mostly because a lowfrequency coil can be made to have a very much lower $r / L$ ratio than a high-frequency coil of the same physical size, as is evident from the fact that the selectivity in cycles of a tuned circuit is proportional to $r / L$, and the fact that low-frequency circuits are much more solective than high-frequency circuits in terms of actual cycles. In practice of course, it would be too complicated to multiply all the way up from audio frequency for example, but a great improvement may be obtained by multiplying from reasonably low frequencies such as the range from 850 to 1000 kilocycles. To illustrate, let us substitute some reasonable values in expression 3. If the fundamental frequency is between 850 and 1000 ke , and a " Q " of 200 is assumed, the ratio $r / L$ is about 30,000 . Taking $g$ as $3000 \times 10^{6}$ and the mean capacitance variation as $1 / 10$ micromicrofarad, the number of cycles shift given by expression 3 is, approximately, numerically equal to the output frequency measured in megacycles. Thus, at 14 Mc the frequency variation would be 14 cycles. This is 14 times better than if the oscillator had been run at a 14 Mc fundamental, using a coil of the same " Q ", and assuming the same capacitance variation and transconductance.

Since the amount of tuming capacitance has not appeared in the expressions for minimum shift, it may be concluded that the stiffness of the circuit is of no importance unless it affects the ratio $r / L$. Data already published indicate that this ratio will be very little different in coils of the same size and shape, but wound for different inductances, using the optimum wire size in each case. Thus, there would seem to be a good deal of latitude in the amount of capacitance that may be used. If the variable condenser is very large, however, it is likely to have large and relatively flexible plates and small clearances, all of which may introduce vibration troubles and changes of the calibration curve with aging. Hence, it does not appear desirable to approach maximum stability by using a "high C" circuit with the tube electrodes connected across the whole circuit, as the amount of capacitance required for maximum stability in this type of circuit may be many thousands of micromicrofarads.

## Practical Circuits

The circuit of Figure 1 was chosen for simplicity of analysis. In practice it is likely to give parasitic oscillations. The same is true of many circuits where grid, cathode, and plate are tapped to the coil at points close together in order to loosen the couplings, as was assumed at the start of the derivation of expression 2 from expression 1. Parasitics may, of course, be suppressed by inserting resistances at suitable points, but this is likely to increase the effective resistance at the desired frequency and hence increase the $r / L$ ratio. It is preferable to use a circuit which does not develop parasitics. Such a circuit is included in Figure 2 which shows the essentials of an exciter that has been in use for some time. The 100 mmf condenser is adjusted until the desired frequency band, 850 to 1000 kc , is just a little more than covered by variation of the 50 mmf condenser after which the


Fig. 2—L is a coil of $\# 24$ wire wound 32 turns per inch an a bakelite tube two and a quarter inches in diameter, the length ol winding being two and three-quarters inches. $T$ is the feed-back or tickler coil and consists of about six turns wound over the grounded end of $L$. An aluminum box contains the entire tuned circuit, grid choke and leak, coil $T$, and the band-pass filtcr.
former is left severely alone so as to keep the calibration of the oscillator unchanged. These two condensers are physically a single unit made for band-spread use, and only the $50-\mathrm{mmf}$ section has a shaft on it. The other data shown in connection with Figure 2 are what are actually in use, but have not been worked out by cut and try to their best values. The whole arrangement is merely illustrative and doubtless could be considerably polished up. In particular the band-pass filter designed to pass with fair uniformity all frequencies between 1700 and 2000 ke could be much improved by experimenting with different damping resistors and varying the coupling between coils. This filter is fixed-tuned so as to avoid any tuning reaction on the oscillator. Incidentally the grid leaks on the doublers really are connected as shown since the amplification constant of these tubes is so high that with the leaks connected from grid to ground the plate currents fall to nothing when the oscillator key is up. By connecting as shown, the plate currents stay up and keep the load on the power supply nearly constant during keying, and also, no r-f chokes are
needed in series with the leaks. As is evident from the diagram, excitation for any band can be obtained by connecting a transmission line to any of the tank links, the only other change when changing bands being that it is well to pull out the tube following the link selected so as to get all the power available from the tank.

While Figure 2 represents the arrangement in use at present, a slightly different scheme for getting the power out of the oscillator, as shown in Figure 3, is believed to be better and avoids the band filter. In Figure 3 the oscillator tube has enough cathode bias to bring the operating point on the steepest part of the grid voltage-plate current characteristic curve in the absence of oscillations. The oscillator tube should be one requiring a large bias for cut-off while the following tube should be one recquiring less bias for cut-off so that with no


Fig. 3
oscillations the plate current of the second tube would be just cut off by the normal bias of the oscillator. Then with even very feeble oscillations the following tube would act as an efficient doubler while with stronger oscillations, that would develop more bias on both tubes by way of the grid leak, the harmonic output of the following tube would be still further increased. The following tube should of course be well screened to prevent reaction of its plate circuit upon the oscillator.

## Conclusions

To recapitulate, in order to obtain the greatest inherent stability:

1. Make the fundamental frequency as low as possible.
2. Make the " $Q$ " of the coil as large as possible at the fundamental frequency. This means that the coil should be as large physically as there is room for within the shield can, subject to clearance of at least half a diameter, as well as that the coil design should be good in other respects.
3. Use the loosest couplings between the tuned circuit and the tube that will give the required output, and use a low enough bias resistor so that the effective transconductance in the oscillating condition is not seriously reduced.
4. Choose for the oscillator tube one which has a high ratio of transconductance to capacilance fluctuations when operating at the required level.
5. Keeping the oscillation strength constant, vary the ratio between the grid and plate couplings. The best ratio depends on the ratio between the capacitance variations of the grid and plate.

After having obtained a good inherent stability, any or all the tricks known to the trade may be added. Some of these are: Temperature compensation in the tuned circuit, or at least arranging this circuit where it will not be heated by the tube or other parts of the transmitter, supporting the tuned circuit on a single rigid member to avoid bending and vibration of its parts, reducing the power taken from the oscillator as much as possible and preferably taking output at a harmonic frequency, supplying screen voltage from a voltage divider whose two portions have resistances chosen to form the combination that best compensates for variations in supply voltage, and stabilizing the supply voltage.

By starting with an oscillator of high inherent stability and then adding the refinements to it, sufficient stability may be obtained for many purposes, even including oscillator keying for output at the highest frequencies used for long-distance communication.

Appendix
Let $x=\frac{C_{P}}{C_{g}}$ and $y=\frac{M_{P}}{M_{g}}$. Then expression 1 may be written in the form $\frac{f}{2} \frac{\omega^{2} M_{g} M_{p}}{L}-\sqrt{C_{g} C_{P}}(1 / x y+x y)$. For any given value of $x$ the minimum value of $(1 / x y-x y)$ that may be obtained by varying $y$ is 2, and this minimum occurs when $y=1 / x$, that is, $\frac{M_{p}}{M_{y}}=\frac{C_{g}}{C_{P}}$.

Furthermore, in order for oscillations to occur, a given grid voltage must cause at least an equal voltage to be fed back, whence the condition for oscillation is approximately $\omega^{2} M_{y} M_{p} \Xi r / g$. Substituting this value of $\omega^{2} M_{g} M_{P}$, and the minimum value of ( $1 / x y+x y$ ) already obtained into the expression above, expression 2 results.

