ELECTRONIC ANALOG COMPUTERS

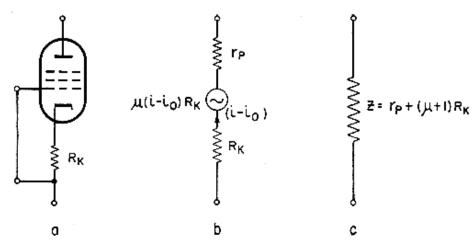
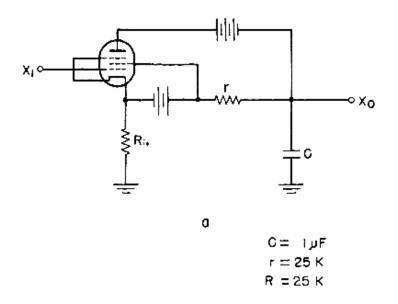


Fig. 4.11. Equivalent circuits illustrating the dynamic plate resistance of a vacuum tube. Because of the current feedback, the circuit tends to oppose changes $(i - i_o)$ in the plate current.



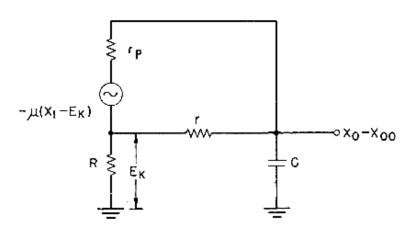


Fig. 4.12. Electronic integrator based on resistance amplification and equivalent circuit.

b

cathode resistor still has a reasonable value. It may be said that the tube has "amplified" the resistance R_K of the cathode resistor. Such resistance amplification is particularly effective with pentodes because of the high effective value of μ .

The application of this principle requires considerable ingenuity, since the tube supply voltages must not be permitted to interfere with the operation of the integrating circuits. In the integrator circuit shown in Fig. 4.12a, use is made of the "dynamic plate resistance" of a tube (triode or pentode) to obtain an "amplified time constant." The equivalent circuit for voltage changes is shown in Fig. 4.12b. Analysis of this circuit yields the transfer function for voltage changes or dynamic transfer function

$$\frac{X_o - X_{o0}}{X_1} = \frac{-\mu}{\frac{r_p}{r} + 1} \frac{1}{\frac{r_p}{R} + (\mu + 1) + \frac{r_p}{r}} RCP + 1$$

$$\frac{r_p}{r} + 1$$
(4.34)

In this analysis, X_{o0} is the value assumed by X_o for $X_1 = 0$; X_{o0} may be referred to as the quiescent, zero signal, or reference output voltage with respect to ground. The time constant b and the gain k/b of this integrator are, respectively,

$$b = \frac{\frac{r_p}{R} + (\mu + 1) + \frac{r_p}{r}}{\frac{r_p}{r} + 1} RC \qquad \frac{k}{b} = \frac{-\mu}{RC \left[\frac{r_p}{R} + (\mu + 1) + \frac{r_p}{r}\right]}$$
(4.35)

In the circuit of Fig. 4.12a, the resistor r shunting the vacuum tube is necessary to provide a path for the d-c plate supply voltage. In order to decrease this shunting effect while still maintaining the d-c plate voltage, McCool² has ingeniously replaced r by the dynamic plate resistance Z of a second tube, as shown in Fig. 4.13. The dynamic transfer function of the improved circuit is

$$\frac{X_o - X_{o0}}{X_1} = \frac{r_p}{\frac{r_p}{Z} + 1} \frac{1}{\frac{r_p}{R} + (\mu + 1) + \frac{r_p}{Z}} RCP + 1$$

$$(4.36)$$

¹ This circuit seems to have been described in the literature first by G. A. Philbrick, Designing Industrial Controllers by Analog, *Electronics*, June, 1948.

² McCool, op. cit.

The high-frequency stability of computer setups like that of Fig. 4.26 will require careful investigation in each case. In general, the loop gain of the feedback loop will vary with the machine variables X_1, X_2, \ldots, X_n . The design of the loop must, therefore, necessarily be a compromise; for this reason, the special implicit methods just described should never be used for accurate computation without the most careful error analysis.

Other Types of Operational Amplifiers. The operational amplifiers hitherto discussed in this section were all of the parallel-feedback type. It is, however, possible to construct operational amplifiers based on series feedback in the manner shown in Fig. 4.27. The integrator circuit of

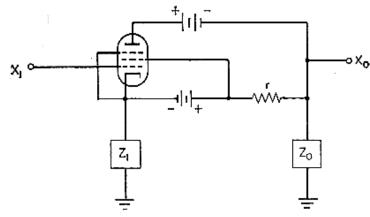


Fig. 4.27 Block diagram of an operational amplifier based on series feedback (G.A. Philbrick, op. cit.), This circuit has been improved by McCool (op. cit.), who replaced the screen resistor r by the dynamic plate resistance of a second pentode (see Sec. 4.4).

Fig. 4.12 is an example of an operational amplifier using series feedback. The circuit of Fig. 4.27¹ has the transfer function²

$$\frac{X_{o}}{X_{1}} = -\frac{Z_{o}(P)}{Z_{1}(P)} \frac{g_{m}}{g_{m} + \left[1 + \frac{Z_{o}(P)}{Z_{1}(P)}\right] \left(\frac{1}{r} + \frac{1}{r_{p}}\right) + \frac{1}{Z_{1}(P)}} \left\{ \approx -\frac{Z_{o}(P)}{Z_{1}(P)} \text{ for } \left|\frac{1}{g_{m}} \left[\left(1 + \frac{Z_{o}}{Z_{1}}\right) \left(\frac{1}{r} + \frac{1}{r_{p}}\right) + \frac{1}{Z_{1}}\right] \right| \ll 1 \right\} \quad (4.75)$$

where g_m and r_p are the transconductance and plate resistance, respectively, of the tube used. The relatively high output impedances associated with series feedback tend to make such circuits less useful for conventional applications than the corresponding parallel-feedback-type operational amplifiers.

¹ Philbrick, op. cit.

² McCool, op. cit.

Finally, it might be interesting to investigate the simultaneous application of parallel and series feedback to operational amplifiers and also the design of operational amplifiers for computers in which the machine variables representing mathematical quantities are currents rather than voltages.

4.7. Regenerative Operational Amplifiers. Reduction of the Effects of Distortion and Instability through the Use of Feedback. One might think that the application of degenerative feedback might decrease the effects of distortion and gain changes and, on the other hand, regeneration could increase the effective forward gain A of an operational amplifier like that of Fig. 4.24b and thus make its performance equation correspond more closely to the desired form

$$X_o = -\left(\frac{X_1}{Z_1} + \frac{X_2}{Z_2} + \cdots\right) Z_o$$
 (4.76)

A closer investigation shows that at least the first of these possibilities has hardly any practical value, as the decrease in forward gain due to degenerative feedback would tend to offset the possible advantages of the degeneration. As a matter of fact, it is shown in the Appendix that, whenever

$$\left| \frac{Z_0}{(1-A)} \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots \right) \right| \ll 1 \tag{4.77}$$

the percentage errors due to distortion and gain changes in the operational amplifier of Fig. 4.24b will be determined by the quantities ϵ_D/A and ϵ_A/A , respectively, where ϵ_D and ϵ_A are the respective percentages of distortion and gain changes in the d-c amplifier used. This implies that percentage errors due to distortion and gain changes in linear operational amplifiers will not be decreased or increased through the application of OVER-ALL feedback (either degenerative or regenerative) around the d-c amplifier used as long as (4.77) holds.

As a result of these considerations, it may be said that a certain amount of over-all regeneration can be used to increase the forward gain of an operational amplifier without directly increasing the effects of distortion and gain changes.

Application of Feedback around Sections of the D-c Amplifier. More interesting results can be achieved through the use of feedback around stages or groups of stages of multistage d-c amplifiers. The amplifier shown in Fig. 4.28 will serve as an illustrative example. It comprises a voltage amplifier section of gain a_1 with a feedback loop of feedback ratio

¹ Korn, G. A., D-c Integrator Design, *Electronics*, May, 1948.