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TIME DOMAIN SYNTHESIS BASED ON A PASSIVE TARDIGRADE MODULE

Background

In this issue we are pleased to present a technique for the analog simulation of linear systems of almost every kind. If you will bear with us patiently you may learn how certain earlier doctrines may be combined with contemporary methods and apparatus to achieve a new and useful end. To some it may seem, indeed, that what we have to say here is largely old-fashioned, but rest assured this is on the whole illusory, and after all *results* are what count. The approach we shall speak about revolves on an approximate time delay element of a somewhat recondite sort, the propriety of which for this purpose is based on the realisation that a completely perfect time-delay element will very probably never be produced. Being willing to accept certain limitations, and not asking for the Moon this year or next, we are rewarded by the practical enjoyment of a tool of considerable convenience today.

The synthesis of systems may proceed from a knowledge of their internal structure, or from evidence of their behavior solely in terms of input and output variables, as in the cases we shall be discussing here. Again, the purposes of synthesis may relate to the nature and performance of the parts within a given single system, or they may be in contemplation of the inclusion of the model of such a system within a larger model of a larger system. It is this latter circumstance we are now concerned with. In other words we are speaking of a *mimic,* to use the language of certain modern physiologists: an apparatus which resembles some prototype entity only as regards the temporal behavior of its input and output variables. It will be taken

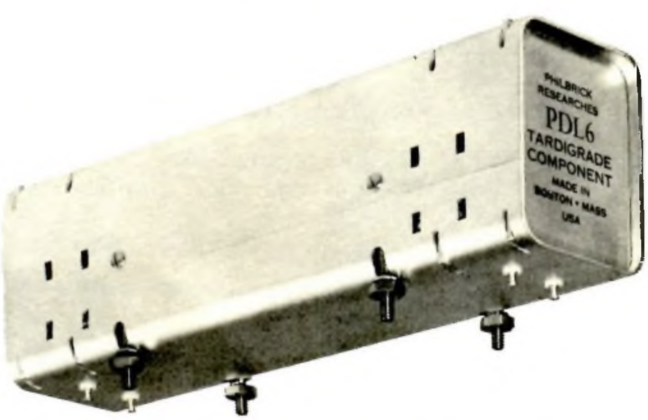

Fig. 1 Standardized Tardigrade Component
GAP/R Model PDL6

as evident that there are many areas of application for an apparatus which has this capability, particularly in analog approaches to system design, and that the utility of such apparatus is dramatically enhanced if its characteristics may be adjusted in a simple manner in the course of adapting it to mimic or represent the behavior of a given prototype. Though not all might consider it so, we count it a big advantage that this present approach to synthesis may be carried out entirely in the time domain.

This kindly attitude toward the time domain, and other attitudes in a similar vein which the reader will encounter if he continues, should not be taken to imply a derogatory prejudice against the frequency domain, which as everyone knows is a very fine domain. The apparatus to be described submits perfectly well to examination in terms of steady state amplitude and phase, as does any linear system. We wish only to make the point that the time dimension is an especially amiable one in the Analog Art. It is further submitted, as humbly as possible, that the passage from linear to nonlinear phenomena happens more naturally to the inhabitant of the time domain, as does also the passage from transient and steady-state to random variations. As a friendly aside to those who think principally in terms of frequency, and who wish dispassionately to compare or to contrast the two domains being considered, it may be useful to transform first from frequency to *period*. For oscillations which are propagated at a constant velocity, period is proportional to wavelength. White light, for example, has uniform energy per interval of wavelength or period, not per interval of frequency, as "white noise" appears to be arbitrarily and somewhat high-handedly defined. But let us return to the central plot.

Linear Signatures in Time

Consider finite, linear, dynamic systems, and the technology of building electronic models thereof. By finite we mean simply that the systems possess fewer than an infinite number of external variables, and generally far fewer. In the case of systems in which the input or output variables are continuously distributed, it is assumed that an adequate representation may be obtained by lumping these variables. By linear we mean that all effects in the systems are proportional to their causes, and that the effects of independent causes are additive. Although no real system is strictly linear, many are usefully approximated as linear; flagrantly nonlinear systems, existing or proposed, will often have subsystems which may be quite accurately portrayed as linear. By dynamic is meant only that the input and output variables may change with time, and of course this usually implies some internal variables in the system which may also be changeable with time. The only simpler systems would be *static* linear systems, but these are altogether too rudimentary — even for this down-to-earth exercise.

Although the method of synthesis to be discussed may be applied to systems with numerous inputs and outputs, let us consider initially that we are confronted by only one of each. One output variable emerges from the system, being causally dependent, through the dynamics of the system, on one input variable. For the purposes already outlined, the system characteristic is uniquely determined by the transient behavior of the output following an isolated and specified change in the input, such change commonly being assumed to have been imposed on a condition of rest. The content of descriptive information in such a transient response, though merely a single function of time, is quite as complete, for example, as that which is offered by the amplitude and phase functions of frequency or period in the case of a sinusoidal test. In an important class of instances moreover, the transient test is easier to make, and in addition amounts to a very familiar experimental action in quite practical environments. It may be well before proceeding to call attention to the importance of certain standard precautions which one should observe in the production of stimulus-response transients. These are outlined in Appendix I below.

In a given primary or prototype system to be synthesized, its time response to an isolated stimulus may for convenience be called its *signature*. Thus we may speak of a *step signature, pulse signature, 4-second ramp signature,* and so on. Specialised branches of engineering employ a variety of terms for this concept, but this term recognizes its generality, while the term response itself seems a little too general. Provided with an adequate signature, the model builder now embarks on a project to synthesize a (linear) model of a (linear) system, based on the naïve but profound doctrine that a similarity of signatures assures a successful forgery. All means are fair in this project, as with synthesis in general, and if elementary juxtapositions or ingenuities suffice, so much the better.

While models may naturally have any physical form, let us consider only electric and electronic types, in which typically the original input and output variables transform into voltages, and in which time ordinarily runs faster or slower than it does in the primary system. On this last point, if the model is a subsystem in a simulator which includes other subsystems which for one reason or another are unchangeable, then the time scale for the model may be most rigorously prescribed, but here we shall assume that the computational task is of the sort in which all scale factors may be chosen for the convenience of the model builder. As to the particular technique to be applied for matching model to prototype, making use of the signature, the model builder will be better able to win out, in the case of most syntheses, if he is given a method and an apparatus for going straightforwardly to his model. One technique to this end, put very briefly, is that of obtaining a weighted summation of the intermediate outputs in a chain of delays. We proceed to develop this technique in more detail.

Synthesis Based on Iterated Delays

Given a perfect linear delay element, which carries an input stimulus continuously and undistorted along a path from which data may be extracted at will, it is theoretically possible to synthesize all linear systems with any desired precision. This continuous method is discussed quantitatively in Appendix IVb, but it is considered to follow intuitively, in entirely practical terms, that any linear system may be approximated by adding the responses, properly attenuated, of a finite sequence of lumped delays. Of course in the limiting case, where the delaying structure is truly continuous and where the weighting factors and the points of extraction are arbitrarily detailed, it is harder (though perhaps not actually impossible) to imagine a workable mechanism. Thus we imagine a useful and practical synthesizing device as comprising a sequence or chain (cascaded, iterated, causally unilateral) of delaying elements of some sort, and a means for obtaining what amounts to a linear combination of the outputs of all the delaying elements in the chain. The modulation coefficients of this linear combination, which may range from a positive maximum through zero to a negative maximum, determine the signature obtained as a response to any given stimulus at the upstream end of the delaying chain, and in consequence characterize the dynamics of the linear system being modelled. This particular sort of delay synthesizer may be called an *output synthesizer*. An *input synthesizer*, on the other hand, results if the input variable is permitted to apply additively, in parallel, to each junction between the delaying elements of the chain, being modulated individually before application by an appropriate coefficient. These two basic forms of synthesizer are shown in Figures 2 and 3.

If all the modulating coefficients referred to above, except the last one in the former case and the first one in the latter case, are set to zero, then one obtains as output the full delay of all delaying elements in sequence. It should be evident that this simpler special case is useful in its own right for the modelling of direct time delays. Its "sharpness", or its approach to an idealized delay, increases as the square root of the number of the delaying elements, assumed isolated and identical, which are placed in sequence. At least as re-

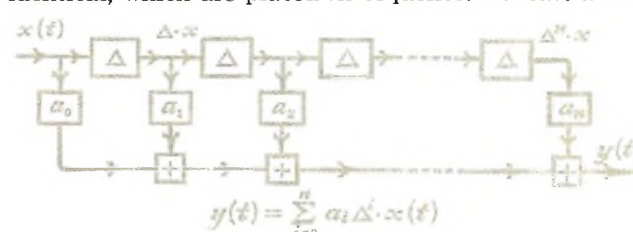

Fig. 2 Output Synthesizer.
The delta-symbol is covered in Figure 5

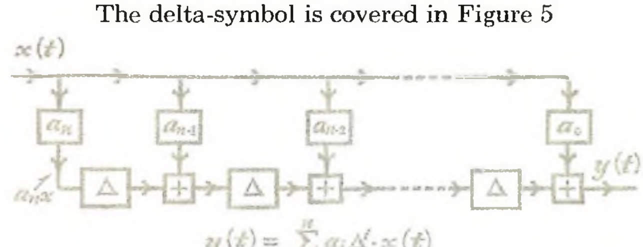

Fig. 3 Input Synthesizer.
Operationally identical to Fig. 2

gards the particular type of delaying element to be *described,* the response of the n-fold delay chain will be substantially as aperiodic, symmetrical, and quiet as that of a single element.

It is contemplated that most linear systems which are waiting to be synthesized will be of the *corresponding* type. That is to say their outputs, in the quiescent steady state, will attain fixed values corresponding in a one-to-one fashion to those of their inputs. It is further supposed that all changes in the outputs will cease within an assignable interval of time following an isolated input stimulus. In those systems which are not embraced under these suppositions, it will most frequently be possible to achieve synthesis by the attachment of standard analog operations in series or parallel with the weighted delay chain structure. A relevant non-corresponding type of system requiring such trickery would be one in which the output may continue to increase even with a steady input; an integrator module has obvious application here. No difficulties are foreseen for "amnesic" systems, namely those in which the output returns to zero eventually for a fixed input level.

As to the delaying elements themselves, these may in the less demanding instances be as simple as ordinary first-order lags, isolated in active sequences. In general, however, more articulate delay elements are in order, and we turn next to a consideration of the elements which are available for this purpose.

Shortcomings of Existing Delays

For the present purposes, most of the well known methods for producing linear time delays of the sharper variety leave a great deal to be desired. They are habitually noisy and unstable, expensive and delicate, unmanageable and temperamental. Some are even noticeably nonlinear, which is a poor way to begin in this particular game. Please understand these stigmata apply to certain of the methods we have used ourselves, and also that in some applications there is no alternative but to live with some awkwardness about the house. For time domain synthesis, however, we can specify what we want and get it, though perhaps it is more effective to enumerate what we do not want. No mechanical moving parts, please, to vibrate or clatter or wear. No AC* methods, thank you just the same, since these will require the modulation and demodulation of a carrier signal. Indeed, no carriers at all are wanted for these synthesizing applications, though they are elsewhere vital and charming. Sampling techniques offer several impressive advantages for delaying structures, in particular that the delay time *itself* may be continuously manipulated, but the sampling process is a notorious source of noise, portending *unpredictable* and annoying strobe interferences. These prohibitions limited the field rather drastically, leaving little else than passive networks arranged along a chain, alternated with and isolated by operational inverters or followers. This approach, which has been used with moderate satisfaction for many years, is most gratifying to manufacturers of amplifiers since potentially it may consume so many of them. But aside from this dubious benefit, with its obvious expense to the user, an excess of amplifiers placed essentially in cascade can lead in this application to an excess of low frequency fluctuation and noise. We are striving in the opposite direction.

The passive networks of the modern analogist are generally constituted of resistors and capacitors; even with a great many of each of these items, accompanied and activated by one or two operational amplifiers, the elemental delay which may be obtained is not an impressive embodiment of a palpable time shift. The density of amplifiers is still rather high. So we come at last, through a highly compressed account of the evolution of the past few years, to an acceptance of the propriety for this purpose of passive networks containing resistance, capacitance, and inductance. This appears at first to take us all the way back into the last century, even to telegraph cables, or at least to Network Analyzers, so-called. Even worse, looking into the classical "delay lines" and other inductive filters, we find at first that the prospects are not so terribly bright. Though the classical inductive filters may be effective for many communications purposes, and even for certain computational ones, we find that they are all prone to ringing, or to overshoot, or to asymmetries of response, or to some other idiosyncrasy not congenial to our idea of an ideal time-domain synthesizing element. Even the formidable theoretical contributions of Chebychev, Butterworth, Padé and others have not turned out to be of ultimate avail in this search. It remained for Dr. H. M. Paynter to find an electric filter which fits our needs quite precisely. This Paynter Delay Line method, as we have come to call it, or PDL for short, was conjured up theoretically but was confirmed and studied here over a period on more than one kind of computing equipment. The basic method itself is an exact one, and is quite general both as to physical media and as to filtering accomplishments. The particular form of PDL which is herewith proposed and offered is described in the next section and elsewhere below. As a delay element for linear synthesis, it has none of the drawbacks which are apparent in alternative structures.

The Paynter Delay Line

The Paynter Line, embodied in the innocent looking networks of Figures 4 and 5, gets results by applying a magic distribution of inductors and capacitors along the filter sections. The theory predicts values for all circuit elements once the choices are made for the average time delay, the characteristic impedance,

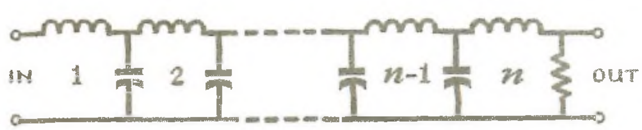

Fig. 4 Orthodox n-section Filter

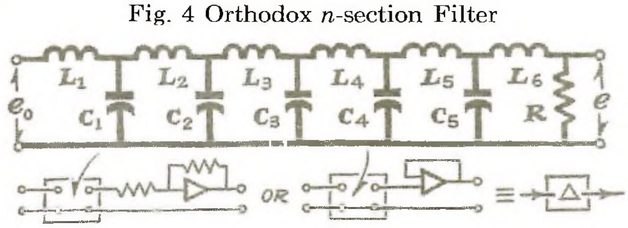

Fig. 5 This 6-section Filter Becomes a Paynter Type if the Inductors and Capacitors Are Properly Assorted

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*That is, not going to zero frequency. What do you suppose this specialised term means to a mechanical engineer say, or to a demographer? Some broader terminology seems in order. Perhaps it is up to systems engineers to supply this.

and the number of sections. For a synthesizer, the latter choice is not merely a compromise between cost of construction and precipitateness of rise. Whereas a very large number of sections would be chosen if the goal were only that of approximating as closely as possible a pure delay, such a delay element would be inappropriate for the present endeavor since few linear systems have signatures with staircase discontinuities. As a time-wise building block we ask not only for a periodicity, flatness of initial response, and symmetry fore and aft, as afforded by the PDL, but also for a

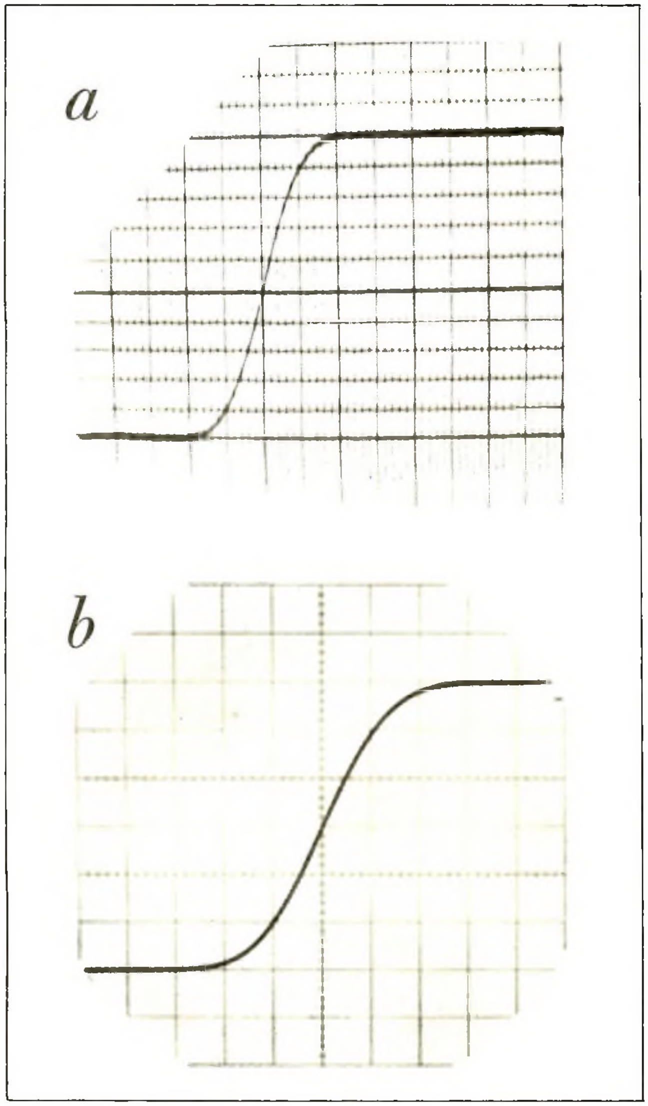


Fig. 6 Response of Single Paynter Delay Component
a. Computed response
b. 1 volt/div. vertical, 0.2 millisec./div. horizontal

rise interval which is on the order of the average delay time, or half-life. This interval is taken somewhat arbitrarily as that from about 1% to about 99% of the rise for a step input. The more traditional 10% to 90% rise time turns out to be nearly ½ the average delay for this case. Six sections, which give the above result, thus serve quite nicely, although more sections or less would also be satisfactory.

An account of the Paynter method, in mathematical terms, is given below in Appendix II. Paynter's general technique is also disclosed in U. S. Letters Patent 3,044,703, filed on June 24, 1954 and issued on July 17, 1962. We should note in passing, as has already been implied above, that filters for delays and other dynamic operations may be constructed according to this method in other embodiments than electric, as is made clear in the patent specification. The inventor himself feels that the method has applications in communication networks, for example, which are of at least comparable importance to those in simulative computational machinery. In our applications, the filter network of Figure 5 is followed by an isolating

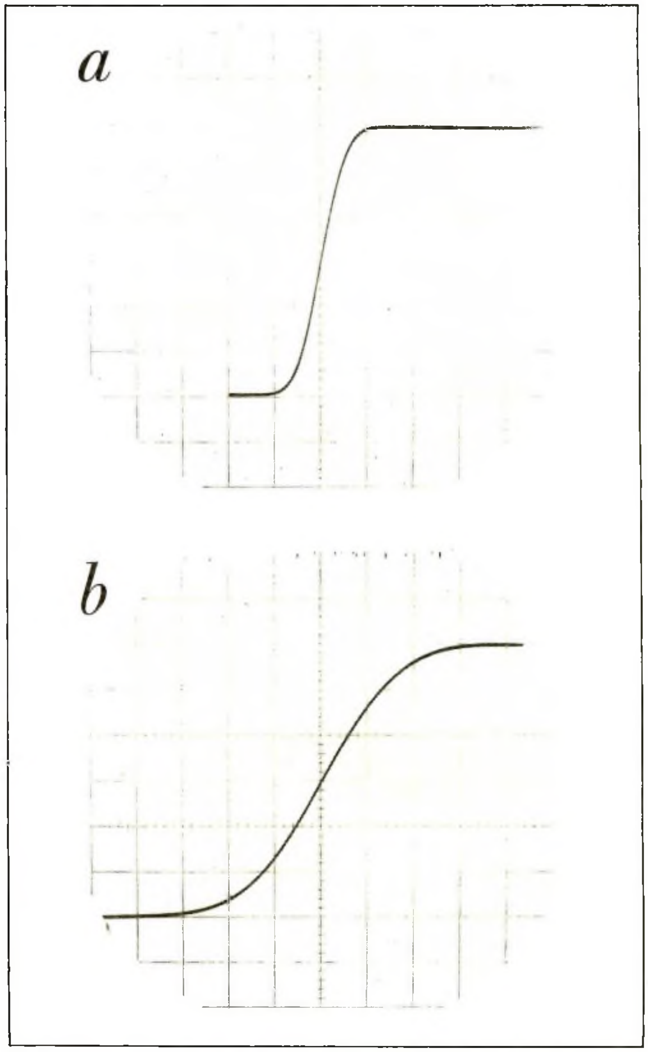


Fig. 7 Response of 10 Paynter Components in Cascade
a. 1 volt/div. vertical, 2 millisec./div. horizontal, time zero at left abscissa
b. 1 volt/div. vertical, 0.2 millisec./div. horizontal, time zero 9 millisec. to left

amplifier as a matter of policy. It should be evident that current through the final resistive member will serve as well for the output variable as does the voltage across it, and that there are many other ways to connect the passive filter element itself into an active amplifier assemblage. We have chosen, in the first delaying, or *tardigrade,* component to be offered for analog synthesis, to set the average time delay at one millisecond, and the characteristic impedance at 10 kilohms. The time response to a step function is shown in Figure 6. Although this component is primarily intended for ±10-volt operation, it may withstand *larger* voltage excursions provided that the final, approximately 10 kilohm, impedance element can dissipate the appropriate wattage. A photograph of the packaged component itself is shown in Figure 1 on the first page. When ten of these components, each followed by its isolating amplifier, are placed in sequence, the rise performance for the whole string is still aperiodic etcetera, though naturally it is relatively more precipitate, while the average delay is now 10 milliseconds. The corresponding response to a step

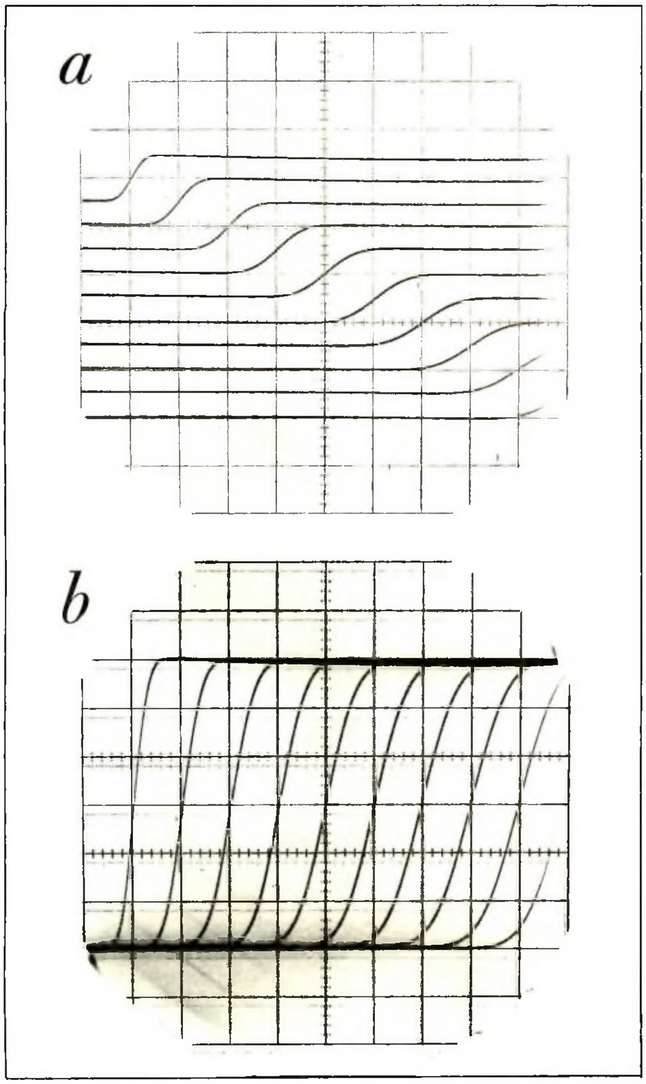


Fig. 8 Response of Each Output in the Cascade of Fig. 7
a. 10 volts/div. vertical, 2 millisec./div. horizontal
b. 1 volt/div. vertical, 1 millisec./div. horizontal

function is shown photographed in Figure 7. Each delaying component taken together with its isolating amplifier may be considered as a convenient *tardigrade module;* when ten such modules are arranged in sequence, the outputs of all modules in response to an input step are as shown in Figure 8. It is evident that the ten-fold delayed response, and consequently any synthesized response built up from any or all of the individual outputs, will be bound to subside to steadiness within a reasonable time after 10 milliseconds. Although when a delay chain of this type, or a synthesizer based thereupon, is *included in* a larger system, as for example when it represents a plant in a regulatory loop, the recovery transients following a perturbation may be sustained for several times 10 milliseconds, nevertheless repetitive solutions of 10 per second or faster are still ordinarily feasible. The delay we have chosen is also considered advantageous, though it is admittedly a compromise to some extent, for use with random perturbations as well as with isolated test stimuli. Futher, in the representation of relatively fast subsystems, within simulators which are operating at "real time" or more slowly, the one millisecond fundamental delay interval is considered to be none too fast. For slower tardigrade components of this same type, the cost will grow roughly in proportion to the average delay intervals. For much slower responses, the cost in relation to that of operational amplifiers will become such that Paynter's active modules with resistor-capacitor networks become more practical. On the other hand there appear to be no practical difficulties standing in the way of producing faster PDL modules, even into and beyond the microsecond realm, for those insatiable analog nanophiles who might wish to seek our help in their breakneck expeditiousnesses. Meanwhile the cost of our present one-millisecond tardigrade component is roughly on the order of that of a good operational amplifier, thus representing an optimum of sorts to a reasonably tolerant economist.

To return to the individual delay component itself, and referring again to the step response shown in Figure 6, considerable effort and devotion have been invested in the smoothness and ideality of its behavior. The inductors and capacitors must be specified to high nominal accuracies, and to low parasitic tolerances, and higher order trims must be included for behavior which is uniform to 3 decimals. Special means must be applied before one can detect a difference in response between individual tardigrade components thus manufactured; high oscilloscope magnifications are required to make any imperfection even visible.

It may be of interest to show here an analog assemblage which has been used to represent and to study the electric filter network of Figure 5. Such an assemblage has been useful in a detailed investigation of tolerances, for example, which may be difficult to predict on purely theoretical grounds. The operational computing assemblage, as shown in Figure 9, offers incidentally a pattern of considerable aesthetic satisfaction, to those who may be susceptible to such incidentals. The analog voltages represent both voltage and current in the inductors and capacitors, and the structure in repetitive operation and display, under direct and convenient manipulation of the values of each circuit element, may be used pedagogically to enable a rapid understanding of this or similar networks as to internal behavior. The responses to an input step, of the nodal voltages inside the filter, are shown in Figure 10. These transients are taken from an analog computing setup such as that shown in Figure 9, in which the values of the circuit parameters were set to those given by the Paynter method. Please note the Philbrick Electronic Graph Display. You may also wish to compare the final response with the "real" filter response of Figure 6. It is assumed that we do not need to explain, to the readers of this journal, how to assemble the computing structure of Figure 9 out of operational amplifiers or more comprehensive

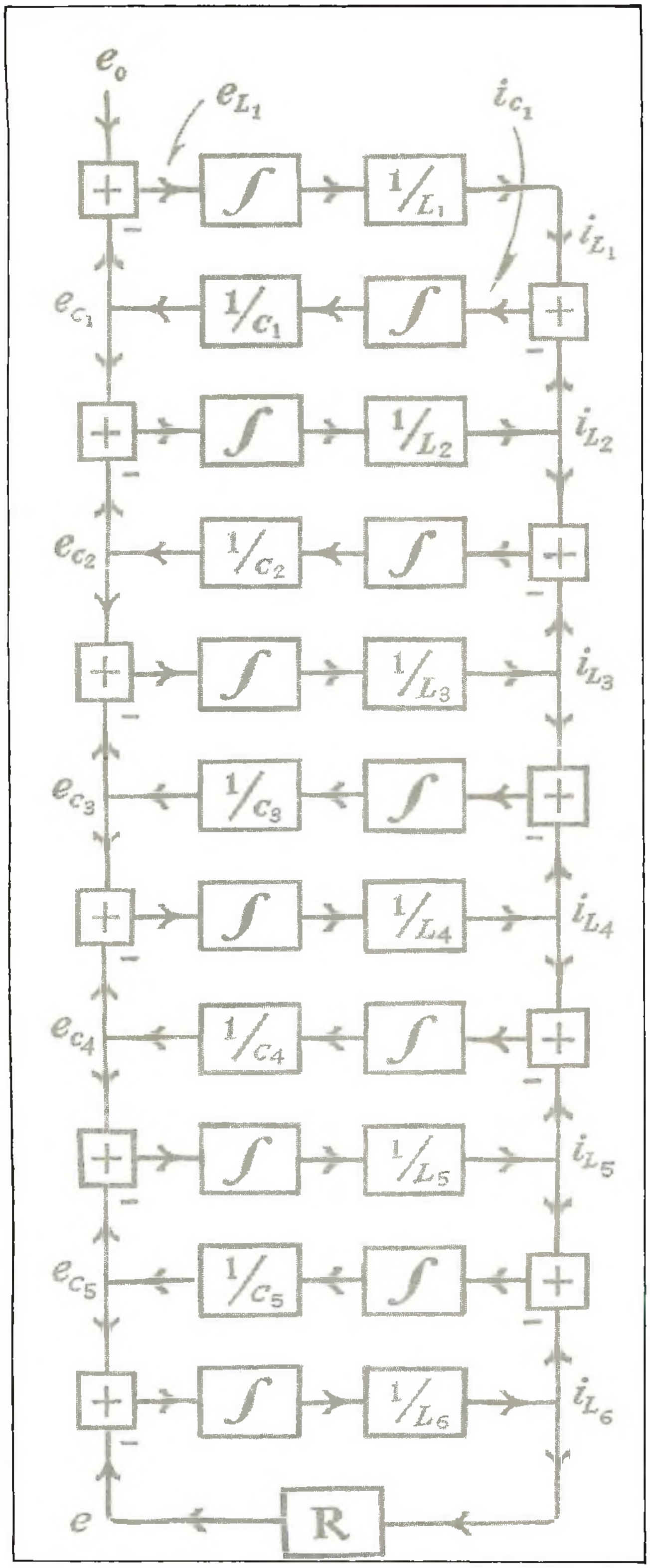


Fig. 9 Analog Representation of the Filter of Fig. 5

analog modules. Actually the earlier GAP/R Model K3 Little Black Boxes have been used for this network representation and study, and more recently the contemporary GAP/R Model SK5-U Universal Linear Operators have also been applied with great local satisfaction all around.

A Short Side Trip without Professor Paynter

The Paynter method is the best we have come across for building passive networks to serve as tardigrade elements for linear synthesis. This method is theoretically exact, astonishingly aperiodic *in view* of and in spite of its complex conjugate roots, if you will forgive this gently academic algebraic divagation, and is apparently unique. Curiously enough, it has been shown in our laboratory, by R. A. Pease, that there is at least one other sequence of choices for the inductors and capacitors of the same class of filter which leads to interesting dynamic results. What we may call the Pease method is one which yields a set of equal real roots, and which is accordingly identical in response to that of a number of identical first-order lags in cascade. To people who have worked here on the passive delay problem this possibility came as a distinct surprise. Some are still unnerved by the fact that a passive chain of dissipationless LC sections may be made aperiodic by a single resistive termination, and that there are at least two assortments of the reactive elements which will achieve this result. A short treatment of the Pease development is given

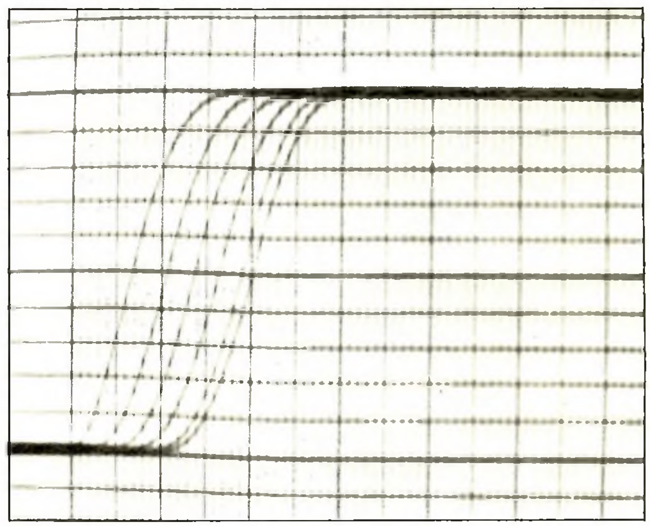


Fig. 10 Computed Responses for the Internal Voltages in the Paynter Filter

below in Appendix III. At one time it was considered that cascaded first-order lags in large number offered the most promising approach to the representation of delayed phenomena. The problem was that so very many amplifiers were required to achieve isolation. It is evident that in the limit, for arbitrarily many such cascades, this sort of elementary structure will approach a perfect delay, as is shown below in Appendix IVa; but the difficulty, of course, is the *rate* at which this approach takes place, and also the lack of fore-and-aft symmetry obtained for only moderately large numbers. The Paynter filter discovery showed the way around this impasse, as it happened, but had it not been for Paynter, the Pease form of the same general kind of filter would have practically irresistible since it eliminates the active isolators which had hitherto been required between low-order lags, and since it is further apparently non-critical. Response photographs for this form, for a 6-section line and under other conditions analogous to those of the response of Figure 6, is shown in Figure 11. Note the longer holdoff in the later reaches of the transient, as compared with the

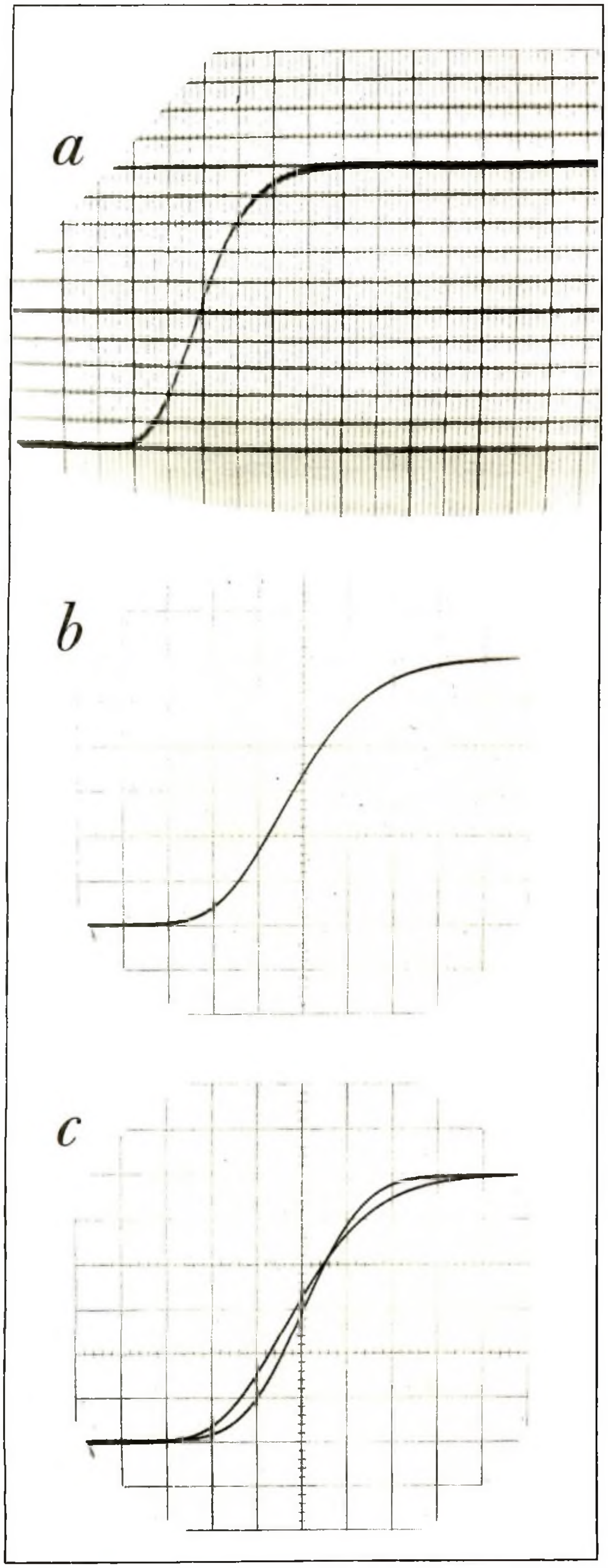


Fig. 11 Response of the Pease Form of Filter
a. Computed response. Compare with Fig. 6*a*
b. Compare with Fig. 6*b*
c. Responses of 6*b* and 11*b* superimposed

corresponding portion of the earlier figure. Although we have not proved it, it might be that this property would be progressively exaggerated in a long iterated assembly, and would then be inimical to the most articulate synthesis.

Applications to Control Problems

There are many more or less obvious applications, in the analog synthesis of plants to be controlled, for approximations to relatively pure time delays such as those which occur when commodities are transported through a duct. For this purpose we propose the usage of iterated modules, having tardigrade components of the Paynter type as described above, in whatever number is appropriate for the character of the delay as shown by experimental means. Beyond this, any plant which resists convenient analysis or standard analog representation may be experimentally modelled, usually with a transformation of the time scale, by means of synthesizers such as those shown in Figures 2 and 3, containing tardigrade components such as those of Figures 1 and 5. Certain kinds of plants, in fact, may be modelled more directly with these structures, or with combinations of them, in a manner which recognizes the similarities between the internal parts of the plant and those of the synthesizers themselves. A ready example is provided by some heat exchangers, and we include counter-current as well as co-current types; experienced counsel on handling such plants is available through our Computing Center. It is predicted, however, that the more general approach, proceeding to simulation of plant properties in terms of input and output behavior alone, will be a more powerful range of application for this kind of synthesizer in the control field.

In a major class of plants which appear to present obstacles to automatic control by simple or standard regulatory mechanisms, part of the difficulty is brought about by the initial inertness of the response of the plant to its manipulated variable, in comparison to the timing of the rest of the response. In view of this it seems likely that in control study applications it will be found to be commonplace that several tardigarde modules, with no coefficients weighting their outputs, will intercede in series within the synthesizer proper. In other words it is very likely for such plants that the "early" coefficients of the synthesizer, namely the first few *a*'s of Figure 2 or Figure 3, will be set to zero. As a matter of fact, the articulation of the early portions of signatures which do not exhibit initial inertness is likely to be rougher than for the later portions, if the arrangement of the synthesizing structure is quite as simple as shown. In those cases in which important fine-structure occurs in the frontal parts of the time responses, it may prove desirable to include smoothing lags of well-known types along with the appropriate weighting channels of the synthesizer. We should like to re-emphasize, however, the importance of the ability of the method here presented to represent very faithfully the complete inertness of initial response in plants which possess transport delays or wave delays. If through imperfect modelling, the analog regulator is permitted to "see through" such delays, the analog experimentalist may work out a fine regulator mechanism which turns out to be meaningless when applied to the real or prototype plant.

Oversimplified but we trust sound, the solution to many strange or difficult control problems is merely:

(a) obtain a signature of the plant observing the precautions already alluded to; (b) model the plant by duplicating the signature on the time scale of the synthesizer, using the same stimulus; (c) with analog components, close the loop with operational controlling devices, starting with standard industrial or servo types, and following up with increasingly esoteric forms of regulatory mechanism, until satisfactory stability and response are both attained; (d) study all relevant disturbing conditions, possibly including random ones, always of course properly scaled to the model, to assure that no one such condition has been favored to the detriment of the behavior under any of the others; (e) compromise if necessary; (f) transform from the characteristics of the optimum regulatory system as found in the model back to the scale of the prototype; (g) build and install the real control mechanism; (h) unless it fails, pretend you dreamed it up out of your intuition; (i) if it does fail, blame it on the synthesizer, but broaden your study next time around. Seriously, it should be pointed out that imperfections such as backlash in valves, which if ignored may lead to disparity between computed and realistic results, may and should be included in the analog model, as part of the plant (sic), of which the synthesizer as described above is then to be considered a linear subsystem. Still further, as a point of view rather than any sort of semantic profundity, one may profitably consider as part of the plant to be modelled, such items as valves, including the remote operating lines which run to them, and the actual measuring elements. In typical cases a new plant may already be under manual control, with local output indication, and local input manipulative means. The "plant" in such an instance, for our present purposes, amounts to the entire dynamic path between these two objects, and the time signature of this plant may be most reliably obtained in terms of this output based on stimulation of this input.

One of the most interesting and fruitful techniques in automatic control is that of including models of the plant as part of the on-line regulatory systems themselves. In analog studies of this technique, two or more synthesizers may conveniently be applied, one for modelling the plant itself and the other for modelling the real model. Among other areas of research which such equipment serves is the study of the effects of mismatch between plants and their regulative models, or of the automatic self-adjustment of such models to suit them to varying plants.

Shown as examples in Fig. 12 are plant signatures based on ten delay modules.

Lest it be considered that the synthesizing technique herein discussed is limited to plants, or to systems in general, which have a single input and a single output, we hasten to point out that multiple plants, that is to say plants with more than one of each of such variables, of the sort in which internal interactions may be blamed for refractory stability problems, may be modelled by means of this synthesizing technique exactly as well as may the simpler variety. Generalization to multiple plants of the techniques of the above paragraphs is straightforward, though it should be pointed out that for a system having m inputs and n outputs, there may be up to mn signatures to be found and synthesized.

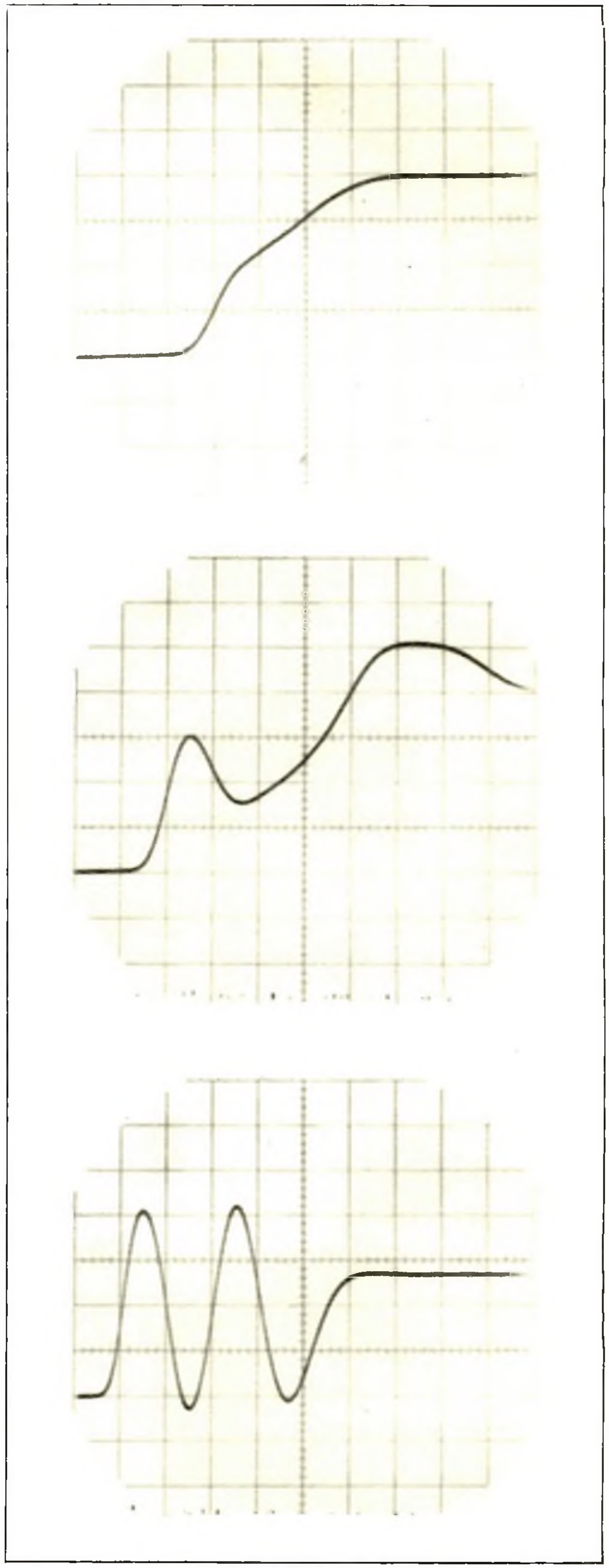


Fig. 12 Typical Synthesized System Responses
(Step-stimulus applied at
left-hand edge of frame)

For systems with more than one input and output, the case of two inputs and two outputs will serve as illustration. Analog synthesis of such systems is similar in spirit to that of the unitary case, being simply more complex; four responses or signatures are obviously involved. Figure 13 shows how a pair of delay chains will constitute the basis for an output synthesizer in generalization of the synthesizer of Figure 2. It is clear that a corresponding generalization may be carried out in terms of an input synthesizer, shown for the simpler case in Figure 3. In a general study of the interactions among the various control loops for plants of this character, and indeed for much more complex cases, the present synthesizer technique has utility not only in modelling the plants themselves, but in modelling the real corrective apparatus which may be applied to circumvent interaction or at least to minimize the instabilities it may lead to.

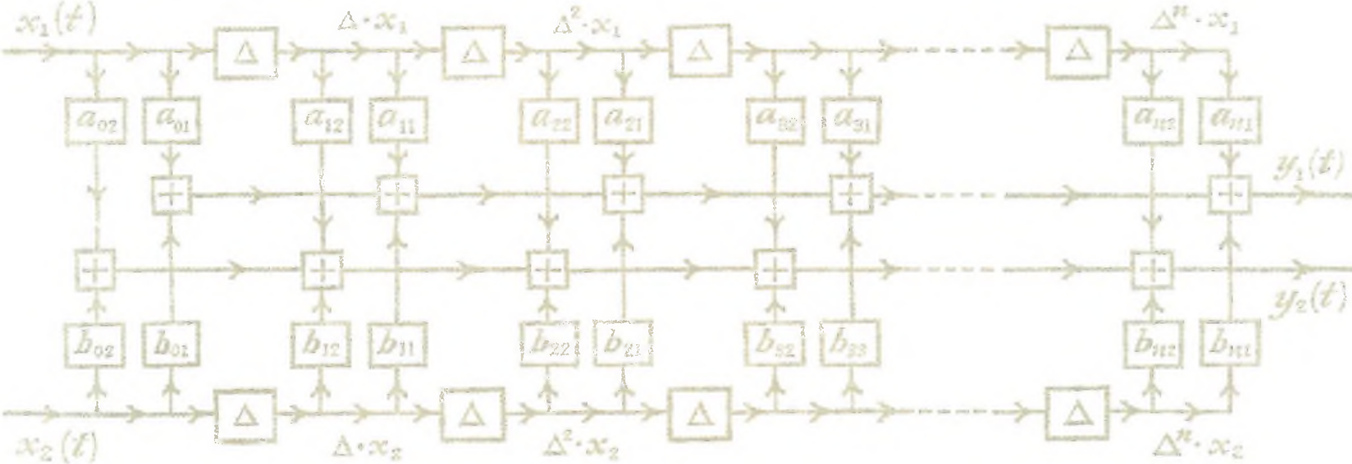


Fig. 13 Synthesizer for Systems with 2 Inputs and 2 Outputs

Appendix I

Precautions in the Experimental Determination of Transient Signatures

(a) Choice of Stimulus

By all odds the commonest stimulus is the traditional step function. This may be fine for corresponding systems, or again for those which always return to a neutral output value for steady inputs, and for systems in which the input may be made to perform a step change in a time interval which is small compared to the first wave of influence through the system. In other cases different sorts of input stimuli must be resorted to. Thus for example to keep the output on scale it may be necessary to apply a square pulse, preferably of relatively short duration, in place of a step. Further it may be more practical to impose a terminated ramp, choosing a rate which the input may faithfully follow. Our main point is that the precise nature of the stimulus is as much a part of the test data as is the response it stimulates.

(b) *Linearity assumptions*

Some assurances should be obtained in advance that the system is reasonably linear, or at least that it is only mildly nonlinear. Symmetry of response for oppositely directed stimuli is a convenient and necessary criterion; it is the least one should expect. Bounds or saturation effects should be religiously avoided, though of course you may wish to simulate these separately at a later stage in the negotiations. Excursions of the input variable should be judiciously limited if they threaten to transcend regions of varied behavior. In such cases the system may be considered as a set of differing systems, one for each significant region of the input variable. Similar remarks apply for other "ambient" conditions. Quite often peripheral nonlinearities which are nothing more than static or scalar effects may be detected by inspection or by simple tests, and may be represented in subsequent synthesis by functions of one (or more) variables in the direct causal path of the plant. In other cases nonlinearity in parallel or mixed paths may be identified, isolated, represented and studied. A bit of detective work is often in order, going backward and forward from analysis to synthesis; every available tool should be brought to bear if you are serious about building a good model. In the most general cases of course, where serious nonlinearities are evident, and where the interior of the prototype is inaccessible (the learning process may stand as an example of this kind), then you have a real problem on your hands, and our best wishes go with you. In the meantime a large group of more docile systems will yield to dynamic representation based on linear synthesizers.

(c) Isolation of Transients

Ideally the test condition will be a peaceful initial equilibrium for both input and output variables, on which to impose an appropriate transient event. Nature is not always quite so kind, and we may need to match our ingenuity to her vagaries. For example, when random variations are present, it will generally be advisable to repeat the stimulus as many times as are required to average out those fluctuations in the response which are not attributable to the stimulus. In many instances a periodic square wave is useful as a stimulus since it affords the opportunity to check on symmetry, and yields the repeated responses which may be superimposed for the averaging process. It is generally important that the half-period of the square wave be as long as the slowest transient in the system under study.

(d) Timing Coordination

One of the most vital steps in transient testing is the accurate location of the moment of application of the stimulus on the recorded response. If this timing identification is not carefully made, it is possible to ignore, for instance, an initial direct time delay in the system. If automatic recording apparatus is applied, an adequately fast chart speed should be provided, and some mark inserted on the record, perhaps automatically, at the instant of inception of whatever stimulus is being imposed.

Appendix II

The Paynter Delay Line

The partial differential equations describing the voltage and current, at any distance x down a uniform lossless electric line, are:

$$l\frac{\partial E}{\partial x} + LpI = 0$$

$$l\frac{\partial I}{\partial x} + CpE = 0$$

In these equations L and C are the series inductance and shunt capacitance respectively for the total line. p is the Heaviside operator $\partial/\partial t$. The equations integrated with respect to x over the line length l enable one to solve for the stimulus voltage E_S and current I_S, in terms of the response voltage E_R and current I_R.

$$\begin{bmatrix} E_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh \tau p & \sinh \tau p \\ \frac{1}{z_0}\sinh \tau p & \cosh \tau p \end{bmatrix} \begin{bmatrix} E_R \\ I_R \end{bmatrix}$$

Where

$$z_0 = \sqrt{\frac{L}{C}} \quad \text{(surge or characteristic impedance)}$$

$$\tau = \sqrt{LC} \quad \text{(transmission time)}$$

When the stimulus end is driven from a low impedance source and the response end is terminated in a resistance R_R, the response end voltage and current are given by:

$$E_R = I_R R_R = E_S/\left(\cosh \tau p + \frac{z_0}{R_R}\sinh \tau p\right)$$

$$= e^{-\tau p} E_S/\left[\frac{1}{2}\left(1 + \frac{z_0}{R_R}\right) + \frac{1}{2}\left(1 - \frac{z_0}{R_R}\right)e^{-2\tau p}\right]$$

This relationship becomes a pure delay if the terminating resistance R_R is matched with the surge impedance:

$$R_R = z_0 = \sqrt{\frac{L}{C}}$$

Then:

$$E_R = e^{-\tau p} E_S$$

Looking into the response end of the line when the stimulus end is short circuited (driven from a low impedance source), the impedance is:

$$z_z = -\left[\frac{E_R}{I_R}\right]_{E_S = o} = z_0 \frac{\sinh \tau p}{\cosh \tau p}$$

The sinh τp and cosh τp terms may be expressed as infinite product expansions:

$$\sinh \tau p = i \sin \frac{\tau p}{i} = \tau p \prod_{k=1}^{\infty} (1 + A_k \tau^2 p^2)$$

$$\cosh \tau p = \cos \frac{\tau p}{i} = \prod_{k=1}^{\infty} (1 + B_k \tau^2 p^2)$$

Where:

$$A_k = \left(\frac{1}{k\pi}\right)^2$$

$$B_k = \left[\frac{2}{(2k-1)\pi}\right]^2$$

Thus:

$$\frac{z_z}{Lp} = \frac{z_0}{Lp}\frac{\sinh \tau p}{\cosh \tau p}$$

$$= \prod_{k=1}^{\infty}\left(\frac{1 + A_k LCp^2}{1 + B_k LCp^2}\right)$$

The Paynter lumped constant approximation for the short circuited uniform lossless transmission line is achieved by matching the first n terms of the product expansion. The n-th degree approximation is:

$$z_{zn} = Lp \prod_{k=1}^{n}\left(\frac{1 + A_k LCp^2}{1 + B_k LCp^2}\right)$$

The numerator and denominator constitute the odd and even parts of a Hurwitz polynomial. This product can therefore be expanded in a finite continued fraction to achieve the impedances and admittances of a passive ladder network such as that shown in Figure 5.

$$z_{z5} = L_6 p + \cfrac{1}{C_5 p + \cfrac{1}{L_5 p + \cdots + \cfrac{1}{C_1 p + \cfrac{1}{L_1 p}}}}$$

The lumped line described above terminated at the response end in its characteristic impedance ($R_R = \sqrt{L/C}$) has the transfer characteristic:

$$E_R = E_S/\left(\left[\prod_{k=1}^{5}(1 + B_k \tau^2 p^2)\right] + \tau p \left[\prod_{k=1}^{5}(1 + A_k \tau^2 p^2)\right]\right)$$

The frequency response characteristics ($p = i\omega$) are:

$$(\text{GAIN})^2 = 1/\left(\left[\prod_{k=1}^{5}(1 - B_k \tau^2 \omega^2)\right]^2 + \tau \omega^2 \left[\prod_{k=1}^{5}(1 - A_k \tau^2 \omega^2)\right]^2\right)$$

$$\tan \varphi = -\tau\omega \prod_{k=1}^{5}\left(\frac{1 - A_k \tau^2 \omega^2}{1 - B_k \tau^2 \omega^2}\right)$$

$$\approx -\tan \tau\omega$$

Since the first eleven poles and zeroes of the tangent function are matched exactly,

$$\varphi = -\tau\omega, \quad \text{for} \quad \varphi = 0, \frac{\pi}{2}, \pi, \ldots, 5\pi,$$

the phase is nearly linear out to 900° of phase shift. This assures that the impulse response is very nearly symmetrical about $t = \tau$. Thus, since the response and the first ten derivatives with respect to time of the impulse response at $t \leq 0$ are zero, the impulse response at $t \geq 2\tau$ will be extremely small and flat.

Appendix III

The Pease Delay Line

A general ladder network driven from a low impedance source at the stimulus end has the transfer characteristic:

$$E_S = M(p)E_R + N(p)I_R$$

and the impedance looking into the response end of the ladder is:

$$z_z = -\left[\frac{E_R}{I_R}\right]_{E_s=0} = z_0\frac{N(p)}{M(p)}$$

Choosing the terminating resistance at the response end equal to z_0:

$$E_R = E_S/[M(p) + N(p)]$$

Let $M(p)$ be an even function of p and $N(p)$ an odd function of p. Then by continued fraction expansion the values of the inductors and capacitors of a finite passive ladder network having the desired response end impedance follow from a continued fraction expansion of

$$z_0\frac{N(p)}{M(p)}$$

identical to that of Appendix II.

For an m-th order equal-lag approximation of the pure delay:

$$\left(1 + \frac{\tau}{m}p\right)^{-m} = M(p) + N(p)$$

$$N(p) = \tau p + \frac{\left(1 - \frac{1}{m}\right)\left(1 - \frac{2}{m}\right)}{3!}\tau^3p^3 + \ldots$$

$$M(p) = 1 + \frac{\left(1 - \frac{1}{m}\right)}{2!}\tau^2p^2$$

$$+ \frac{\left(1 - \frac{1}{m}\right)\left(1 - \frac{2}{m}\right)\left(1 - \frac{3}{m}\right)}{4!}\tau^4p^4$$

$$+ \ldots$$

$$\tau = \sqrt{L/C}, \quad z_0 = \sqrt{\frac{L}{C}}, \quad m = 2n - 1$$

Appendix IV

Some Formal Operational Considerations

A. Proof that a cascade of n equal lags approaches a pure delay as n approaches infinity:

The unit delay is characterized by the operator e^{-p}. Its logarithm is $-p$. The n-fold cascade of lags, for unity cumulative delay, is characterized by the operator:

$$\left(1 + \frac{p}{n}\right)^{-n}$$

Its logarithm may be expanded in a convergent power series provided $\left|\frac{p}{n}\right| < 1$.

$$-n \log\left(1 + \frac{p}{n}\right) =$$

$$-n\left[\frac{p}{n} - \frac{1}{2}\left(\frac{p}{n}\right)^2 + \frac{1}{3}\left(\frac{p}{n}\right)^3 - \ldots - \frac{1}{m}\left(-\frac{p}{n}\right)^m + \ldots\right]$$

$$= -p\left[1 - \frac{1}{2}\frac{p}{n} + \frac{1}{3}\left(\frac{p}{n}\right)^2 - \ldots + \frac{1}{m}\left(-\frac{p}{n}\right)^{m-1} - \ldots\right]$$

$$\log\left[\lim_{n\to\infty}\left(1 + \frac{p}{n}\right)^{-n}\right] = \lim_{n\to\infty}\left[-n\log\left(1 + \frac{p}{n}\right)\right] = -p$$

Therefore provided $|p|$ is finite:

$$\lim_{n\to\infty}\left(1 + \frac{p}{n}\right)^{-n} = e^{-p}$$

B. If one had a semi-infinite, continuous, and perfect delay line and could accumulate a continuously weighted version of the signal at every point down the line following a pulse input, the response would be

$$\int_0^\infty f(g)e^{-gp}\,dg \qquad \text{I}$$

where g is the distance down the line measured in time units, and $f(g)$ is the weighting function.

If now $f(t)$ is also the pulse response of a linear system, then the above weighted delay structure is an exact model of that linear system, and I is its operational representation. If the pulse response of the system vanishes after a time T, then I is simply

$$\int_0^T f(g)e^{-gp}\,dg$$

since

$$\int_T^\infty f(g)e^{-gp}\,dg = 0$$

CORPORATE NEWS

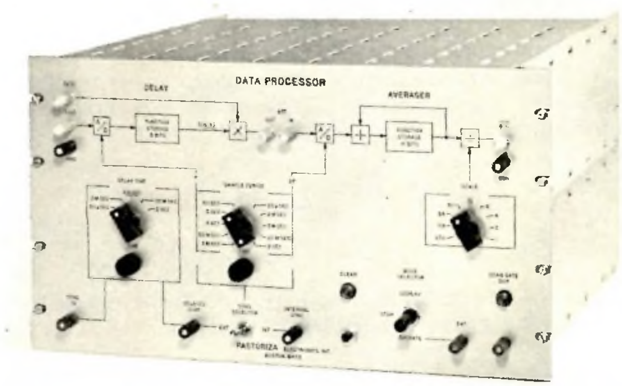
Announcement of the acquisition of PASTORIZA ELECTRONICS, INC., of Boston, Massachusetts, effective 1 July 1963, is made jointly by George A. Philbrick, President of Philbrick Researches, and James J. Pastoriza, President of Pastoriza Electronics.

This addition to Philbrick's corporate family was purchased for an undisclosed amount to extend the facilities and capabilities of both companies. The two principals have collaborated for a number of years in the design and development of electronic devices which occupy areas of mutual interest. Pastoriza himself is scheduled to become Research Director, as well as a Vice President of Philbrick Researches.

The addition of Pastoriza Electronics will lead to greater efficiency for supplying customer's needs for special-purpose instruments and equipment that are not standard in the existing Philpbrick product line.

Pastoriza has developed a broad product line in analog-digital processing and computing equipment (two representative units are shown in the acccompanying photographs) and in new analog teaching devices.

Pastoriza Electronics have also been consultants on, and designers of, data processing equipment in diverse fields, using analog techniques for voltage handling together with digital conversion and storage. They have designed on-line control instrumentation for an impressive list of customers. For the military services they evolved a Satellite Simulator which digests information from IBM apparatus to track and predict the orbital paths in real or accelerated time.

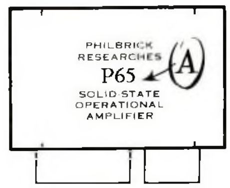


Contributions in medical instrumentation include an Intracardiac Oximeter for Bellevue Hospital (See Vol. II, No. 2 of The Lightning Empiricist), and an Auto-Correlator for Massachusetts General Hospital used in the study of brain defects and recently applied as electrotherapy for the arresting of epileptic seizures.

For MIT researchers Pastoriza designed and built a Diffraction Grating Analyzer, a Contour Display of Magnetic Fields used for study and actual degaussing of naval vessels, a Magnetic Controller for Amplidynes that has an auxiliary device which senses fluctuations in magnetic fields, and a number of other items.

PRODUCT NEWS

In the inexorable march of progress, Model P65 Differential Operational Amplifier, described at length in the April issue of this Journal, has given way to Model *P65A*. The "A" version has twice as much output (2 milliamperes at ±10 volts) and nearly half as much thermal voltage offset (6 millivolts guaranteed maximum — −25°C to +85°C).



P65's tough little twin, PP65, has also grown into *PP65A*, with similarly improved performance.

Progress and improvement have also infected the 6151, now called *P75*. P75's minimum voltage gain is 20,000 maximum thermal shift (−25°C to +85°C) 12 millivolts, input impedance (dc) 5 - 10 megohms *between* inputs, and better than 100 megohms from either input to ground. It is at least 10 - 20 times better than P65A as a long-term integrator in HOLD.

Newly added Models *P55A* and *PP55A*, low cost utility cousins of P65A and PP65A, use hermetically-sealed silicon semiconductors throughout, have half the gain of P65A, thrice the voltage and current drift, equal output current. Operating temperature range is comparable and expected reliability is perhaps even greater in P55A, because, like PP55A, it contains no bias adjustment potentiometer. If needed, it can be connected externally. This publication's non-commercial nature inhibits mention of price, but our Sales Department or your local Philbrick Representative will readily tell you that in quantiy these Models are even lower in cost than corresponding thermionic types.

Briefs: A holddown for P65A and its ilk is now available . . We are about to use a new socket having twice as many contact areas as that presently used for P65A . . The new socket is still the reliable accordion type, firm, gentle, and capable of a good many more insertions and withdrawals than are likely in most applications . . The electro-mechanical chopper used in Model SP656 is guaranteed for two years . . The same chopper, available at a premium, will behave nicely for long intervals in thermionic Models, such as K2-P and USA-3 . . Our PP Models, embedded in epoxy forever, are evacuated during potting, and should be capable of braving the vacuua of outer space.

The following new publications are available:
- Applications Brief D5: "P65 as *Amplifier* with High Input Impedance"
- Applications Brief D6: "Operational Amplifier as Direct-reading Precision Resistance Comparator"
- Applications Brief D7: "Operational Amplifier as Constant-Current Source — I"
- List of Available Reprints
- Solid State Amplifier Comparison Chart

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