THE LIGHTNING EMPIRICIST

Advocating electronic models, at least until livelier instrumentalities emerge

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IMPEDANCE & ADMITTANCE TRANSFORMATIONS using Operational Amplifiers

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I. Transadmittance, Transimpedance, Positive and Negative Self-Impedance through Active Circuits, including references to:

Photomultiplier and ion-current amplifiers;
Current sources and generators; and
Negative resistors and capacitors for dynamic compensation with Single-Ended, Differential, and Inverted Amplifiers

This is to be an abbreviated contemplation of a subject, the rigorous or exhaustive treatment of which might occupy a veritable volume. Our modest ambition here is to present a few unusual principles, with possible circuit applications, which might stimulate the thoughtful Reader to increase his rate of consumption of operational amplifiers.

INVERTING AMPLIFIERS

The conventional ideal inverting circuit is often simply described as one which maintains a voltage null and a current balance at its summing point, or negative input terminal, by feedback around a dc amplifier having high *power* gain. Such a circuit is exhibited in Figure 1. It is immediately seen that if the voltage null and current balance are well maintained, the output voltage is determined solely by the total input current and the feedback impedance; and the input current is the sum of the independent input currents, each determined solely by its input voltage and impedance. The first order effects of discrepancy between theory and practice are described elsewhere.

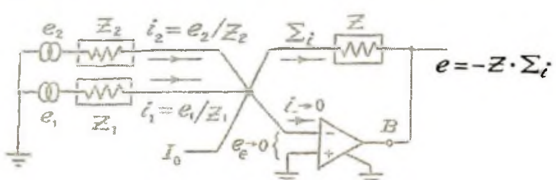

Fig. 1 Ideal Single-ended Operational Amplifier

These relationships immediately suggest, to one attuned and vigilant, that current from a current source, or charge from a charge source, can be converted into voltage at a specified level by proper choice of feedback impedance, and that an input voltage can be used to establish the current through an element connected in the feedback path of the amplifier. At first blush one might think of the circuits of Figure 2, briefly described as follows:

In Figure 2a, a photomultiplier, an ion-producing circuit, or some other high-impedance source of current feeds its current directly into (or more often, sucks the current i directly from) the summing point of the amplifier. This is not cruel when one remembers that current sources prefer to deliver their current to voltage nulls, which appear as low impedances and which do not load. In the example shown, an additional large resistor R_0 establishes the magnitude of an opposing current. I_0, which may furnish an index level or threshold, or compensate for "dark current" or background. The sum of the currents flows through the feedback element and produces an output voltage e.

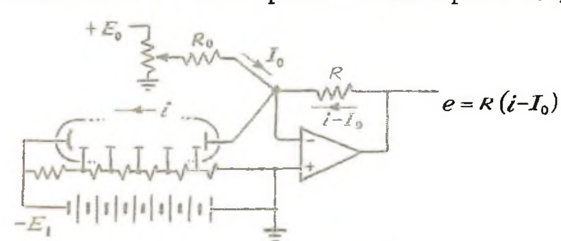

Fig. 2a Current-to-Voltage Transimpedance

The feedback element can be a linear resistor of virtually any magnitude for direct measurement of current; it can be a nonlinear resistor to produce a nonlinear function of current, thus compensating for nonlinearity in the transducer; or it can be a capacitor, to measure the integral of the input current, interpreted directly as charge, or of course the *average* input current over a given period. Because current is converted to voltage, usually substantial, the major error is current leakage, which determines the choice of amplifier. [Sales Manager's Note: P2 is considered an excellent choice.] Excitation voltage, E_1 is quite high, sometimes kilovolts; output is generally in volts; and erroneous millivolts or microvolts at the input e_ϵ produce millivolts or microvolts at the output, because voltage amplification is small. Common mode effects are

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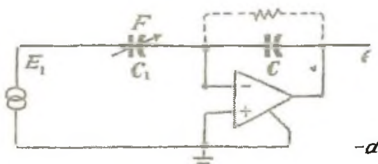
negligible, because the summing point is maintained close to zero. Voltage noise in the external circuit can be kept out without difficulty, because it is feasible to use grounded guards on the summing point without appreciably affecting response time. This approach to measurement contrasts favorably with the "potentiometric" approach, wherein (small) voltage is developed across a resistor, whose size is limited to avoid loading. The voltage, in being amplified, must cope with amplifier voltage offsets, common mode swing, and common mode circuit impedances, as well as voltage noise in the external circuit. [Sales Manager's Note: If you insist on the potentiometric approach, P2 is still an excellent choice, because of its low noise power, lower common mode rejection, less than 5×10^{-11} ampere offset current, and an even lower — 10^{-12} ampere — variation in that current.]



$$Q = C_1 E = -Ce$$
$$dQ = -C\,de$$
$$-de = E_1 \frac{dC_1}{C} + \frac{C_1}{C} dE_1$$

Fig. 2b Measuring Capacitive Sources (Charge-to-Voltage Transimpedance)

In Figure 2b, an input capacitor is subjected to a varying stress. The operational amplifier preserves charge variation in the feedback element, and the output voltage measures the charge or the change in charge. In the case of the capacitive transducer (e.g. microphone), a fixed voltage stress E_1 is applied, and ΔQ is proportional to the change ΔC_1 in capacitance resulting from motion. Note the important properties of the operational amplifier that are especially useful in this application: the summing point is held at ground potential, hence voltage across the capacitor is truly constant; ac coupling is inherent, hence the dc excitation voltage is kept off the summing point; and the capacitance of shielded leads between the transducer and the amplifier does not significantly load the transducer, because that capacitance appears between summing point and ground.

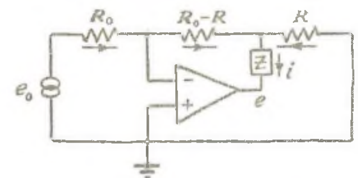
Question for the Reader to consider: Can one treat a piezoelectric transducer in this fashion? One might reason that the fixed stress (or "bias"), instead of being electrical, is mechanical, energy being stored in a spring instead of a battery, but that the basic response to stress is still a capacitance change, and hence change of charge, or a temporary flow of current.

In the case of the capacitor comparison bridge, there is no mechanical stress applied to the unknown capacitor C_1. The stress is electrical, in the form of an ac voltage e_1. An ac voltage is produced at the amplifier output, proportional to the ratio of input to feedback capacitance. A pair of equal external resistors can be used to provide a null measurement.

The principal drawback of the operational amplifier approach is that the dc leakage current will charge the feedback capacitor and eventually cause the amplifier to reach saturation. However, this possibility can be forestalled by using a large parallel resistor as a "leak" to establish the maximum voltage to which the output can depart. Note also that potentiometric approaches have the same "open grid" problem.

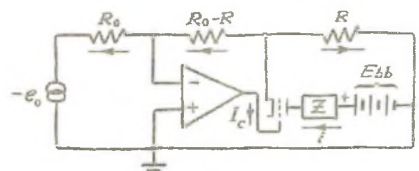
[Sales Manager's Note: Models P75, PP35A (especially), and P2 (for low frequency applications where capacitors are large and leakage important) are suitable for forward-looking devotees of solid state, noting the P2 and PP35A will serve potentiometrically as well. For those who still believe in thermionics, Models K2-W and SK2-V will do the job economically and well.]

In Figure 1, we saw a current generator perform the simplest job of its kind. The output voltage is adjusted by the amplifier to maintain whatever voltage is needed across the load element Z to keep the summing point at zero, or at a *virtual ground*. In this circuit, the voltage source is grounded, but the load cannot be grounded in the literal sense. The amplifier is single-ended: neither input nor output is used differentially. There is no net amplification of current, and the sources and the amplifier must both supply the current in Z. Any number of grounded sources may be used, and this current will be equal to the sum of the input currents. A current and voltage booster may be used at the point in the circuit marked "B" to provide any needed degree of augmentation. [Sales Manager's Note: Philbrick manufactures a number of popular boosters, and can advise on boosting circuitry in any case.]



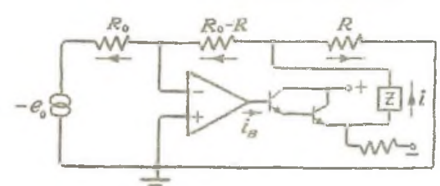
$$i = \frac{e_0}{R}$$
$$e_{MAX} = -i_{MAX} Z_{MAX}\left[1 + \frac{R(R_0-R)}{R_0 Z_{MAX}}\right]$$

Fig. 3a Voltage to Current Transconductance



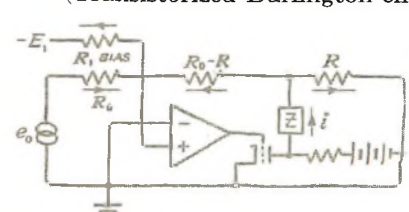
$$i = \frac{e_0}{R} \quad (I_c \lll i)$$

Fig. 3b Boosted Voltage to Current Transconductance (Plate-loaded)



$$i = \frac{e_0}{R} \quad (i_B \lll i)$$

Fig. 3c Boosted Voltage to Current Transconductance (Transistorized Darlington emitter follower)



$$i = \frac{E_1}{R}\frac{R_0}{R_1} - \frac{e_0}{R}$$
$$\frac{\partial i}{\partial e_0} = -\frac{1}{R}$$

Fig. 3d Boosted Voltage to Current Transconductance (Shunt-controlled current)

In Figure 3, we see a number of possibilities for single-amplifier current generators using grounded sources. These circuits all have current amplification, both terminals of the current-receiving element (or

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ANALOG TECHNIQUES APPLIED TO BUSINESS MODELS

F. M. Verzuh* and P. D. Hansen†

Economics in the Small Business

The business executive has become increasingly concerned with the broad economic aspects of the product he manufactures. He is no longer able to confine his attention narrowly to the immediate basic cost of the running of his business. Rather, he must be aware of the many different economic factors which affect his whole operation. Rapid advances in product technology, improved communications among customers and competitors, and shorter business and economic cycles demand management decisions which are prompt and accurate.

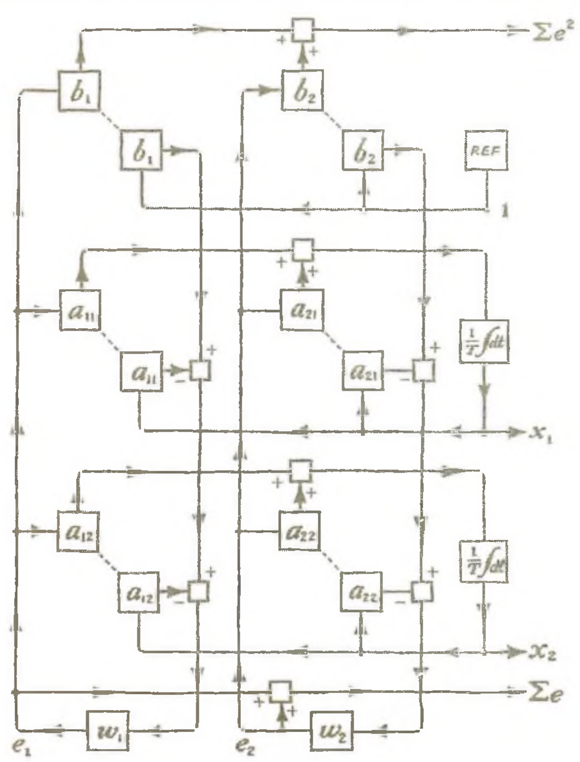
Economic phenomena previously considered remote from the daily or weekly operation of a business are now important to it. The need for awareness is now so great that the government releases numerous frequent publications in this area. For example, the Commerce Department releases *Business Cycle Developments*, a monthly report containing about 70 leading indicators and over 300 economic components along with various measures of their importance and interrelationships. The newspapers feature a continual barrage of articles which discuss the current behavior of various economic variables. "New Construction Falls for Second Month"; "Instalment Credit Continues to Rise"; "President Concerned if Economy Will Remain Strong in 1964"; are typical headlines of items in The Wall Street Journal. Since many of the "leading indicators" are contradictory, the reader may understandably be confused. What are the relevant relationships among the variables? What effect will they have on the future of one's business? Is 'New Housing Starts' a good business indicator for the building industry? What are the economic variables affecting a particular market?

Analog Models

The high-speed capabilities of general purpose analog computing machines, for solving simultaneous differential equations, have been long been exploited by engineers and scientists in their studies of the design and behavior of dynamic physical systems. The effect of an experimental change in a system is computed in small fractions of a second. This speed, along with a graphical display, assists the operator in devising subsequent experiments. For the most part, however, those capabilities have been hard to realize in the business field. Realistic dynamic models of business or economic systems are often very difficult to formulate and once formulated may be too complex to program on an analog machine of reasonable size. Furthermore, the recorded data characterizing the interactions between an existing business system and its environment is often fragmentary, and may contain random components of long-period compared with the time span of the data. Experiments in the real world to verify a dynamic model are usually awkward to control, and may even have unpopular effects.

Instead of building dynamic models in terms of continuous differential equations, numerical methods for analyzing discrete or sampled time series are often employed to construct static or quasi-static mathematical models of business and economic activity. These numerical methods typically require solution of redundant simultaneous linear algebraic equations for parameters which provide a best fit of some hypothesized equation (model) to the data. This procedure is called multiple linear regression. Solution with a desk calculator, following for example the iterative Gauss-Seidel process, is very time consuming.

Much money and effort has been expended in the development of digital programs for multiple linear regression. Indeed problems with more than 100 unknowns have been solved with these programs. Even though the actual computing time may be only a few minutes, the input, compute, output (or through-put) time is often a matter of hours or days. As a result the user is not able to exercise creative judgment and reasoning during the research investigation. Too often such a user discovers he should have obtained many additional parametric solutions and that some of those he did achieve are of little interest. These difficulties could have been avoided had the investigator had access to the results as they were achieved, and the ability to control future runs.



Block Diagram
for 2×2 Asymmetric Matrix $(a_{ij} \neq a_{ji})$

$$T\frac{dx_j}{dt} = \sum_{i=1}^{2} a_{ij} e_i$$

$$e_i = b_i - \sum_{j=1}^{2} a_{ij} x_j$$

$$\sum e^2 = b_1 e_1 + b_2 e_2$$

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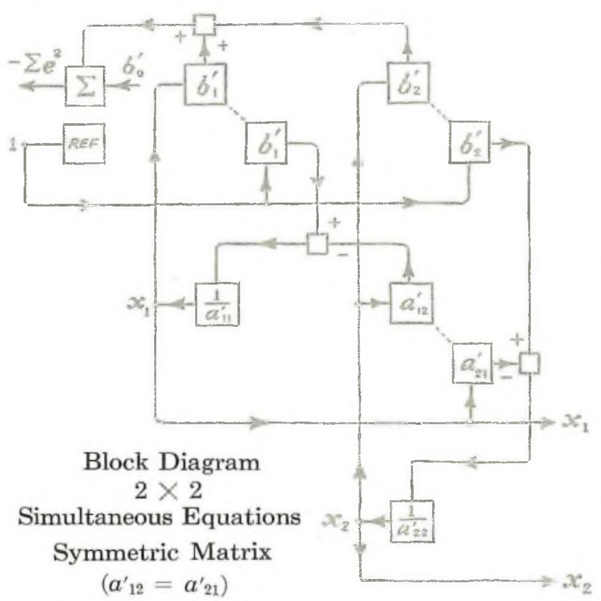
It is for these reasons that one is tempted to solve simultaneous algebraic equations on an analog computor. After all, a solution for algebraic equations may be thought of as the steady-state solution of some set of ordinary differential equations: the *forte* of analog computors. Finding a suitable set of differential equations is not a trivial problem, however, since the solution must be stable for a steady state to exist. Nevertheless, a well known method, which is summarized in the Appendix, is capable of stable attainment of the solution whenever a solution to the original algebraic equations exists.



Block Diagram
2×2
Simultaneous Equations
Symmetric Matrix
$(a'_{12} = a'_{21})$

$$x_1 = \frac{1}{a'_{11}}\left[b'_1 - a'_{12}\,x_2\right]$$

$$x_2 = \frac{1}{a'_{22}}\left[b'_2 - a'_{21}\,x_1\right]$$

$$-\sum e^2 = b'_0 + b'_1\,x_1 + b'_2\,x_2$$

Schematic block diagrams for the almost trivial problem of solving two equations for two unknowns are shown in the adjacent figures. The method and the equations are developed in the Appendix. Note the complex but regular interconnections, and note also that (almost) every coefficient appears twice. Whereas these properties can be exploited in a special purpose analog computor, they make solving even a modest number of algebraic equations awkward with general purpose equipment.

We plan soon to reveal a special purpose analog machine suitable for these three types of problems:

a) Linear Algebraic Equations, including Matrix Inversion and Multiplication
b) Linear Multiple Regression
c) Linear Programming

These are to be solved in a continuous dynamic manner. Transients subside in less than a millisecond; in the cases having symmetric coefficient matrices subsidence takes less than ten microseconds. Some other features include:

(1) Numerical coefficient settings as introduced in our K5 equipment, in which discrete and tandem values are rapidly adjustable and discernible in a convenient matrix array.
(2) Complete modular construction allowing arbitrary expansions, yet with no external patching. Interconnections are committed by the row and column position of the module in the matrix array.
(3) All equipment is solid state, having small power requirements, low thermal dissipation, compactness and reliability.
(4) Low cost compared with general purpose analog and digital equipment.

For purposes of visualizing the way this equipment solves algebraic equations, we refer to the much simplified spatial diagrams here shown. Interconnections similar to those of the earlier diagrams are implied, as

b_1	b_2	b_i	b_m	Σe^2
a_{11}	a_{21}	a_{i1}	a_{m1}	x_1
a_{12}	a_{22}	a_{i2}	a_{m2}	x_2
a_{1j}	a_{2j}	a_{ij}	a_{mj}	x_j
a_{1n}	a_{2n}	a_{in}	a_{mn}	x_n
e_1	e_2	e_i	e_m	Σe

$m \times n$
Simultaneous Equations
m equations
n unknowns
See Appendix Equations 1, 2 and 3

$-\Sigma e^2$	b'_1	b'_2	b'_{k1}	b'_n	→ Σe^2
	$\frac{1}{a'_{11}}$	a'_{21}	a'_{k1}	a'_{n1}	→ x_1
		$\frac{1}{a'_{22}}$	a'_{k2}	a'_{n2}	→ x_2
			a'_{kj} $\frac{1}{a'_{jj}}$	a'_{nj}	→ x_j
				$\frac{1}{a'_{nn}}$	→ x_n

$n \times n$
Simultaneous Equations
Symmetric Matrix
$a'_{jk} = a'_{kj}$
See Appendix Equations 4 and 5

	b_1	b_2	b_i	b_m	Σe^2
c_1	a_{11}	a_{21}	a_{i1}	a_{m1}	x_1
c_2	a_{12}	a_{22}	a_{i2}	a_{m2}	x_2
c_j	a_{1j}	a_{2j}	a_{ij}	a_{mj}	x_j
c_n	a_{1n}	a_{2n}	a_{in}	a_{mn}	x_n
P	e_1	e_2	e_i	e_m	Σe

Linear Programming
$m \times n$
Constraint Matrix
See Appendix Equations 6, 7, 8, and 9

is the use of dual (or tandem) coefficients. Changing from one type of problem to another is accomplished by inserting modular units into appropriate locations as indicated by the block diagrams.

The requisite accuracy of the solution for mathematical modelling is often grossly overstated. Often, the accuracy of the available pertinent input data used in the model is such to preclude all justification for the use of 8 decimal digit arithmetic on digital machines. 3 digit analog accuracy is ample for the data used in most mathematical modelling problems. Furthermore the accuracy of the analog solution can be improved without limit by iteration, adding at least 2 significant figures each cycle.

The availability of low-cost personalized analog equipment of this sort will provide the user with a means for identifying the basic problem, for isolating the relevant factors, for tracing the relationships among variables, and for formulating decisions relating to the model. Furthermore, the analog may be used to simulate the system, to compare observed reactions, to revise the model if necessary, and to experiment with critical parameters in the system.

There are many areas in which the special purpose analog machine may be effectively applied. We have studied mathematical models of the following types:

a) Agriculture — Factor affecting the supply and prices of hogs.
b) Automobile Industry — Factors affecting the demand for and sale of automobiles.
c) Building Industry — Factors affecting the number of new housing starts.
d) Business Models — Factors affecting the rate of industrial production.
e) Retail Sales Models — Factors affecting the sales of certain consumer goods.
f) Stock Market Models — Factors affecting the share volume of the New York Stock Exchange.

Certain studies were made using both analog and digital computing machines on the same mathematical models. The results of these comparative investigations have provided a realistic basis for evaluating the advantages and limitations of the two types of machines for the solution of such problems.

Linear Programming

The algebraic analog is capable of solving linear programming problems with little more difficulty than in solving simultaneous algebraic equations. Although longer solution periods are involved, nevertheless subsidence times are generally less than 10 milliseconds. This is an important capability in the field of operations research.

A typical application involves blending of various substances, each having a specified per unit cost and composition. The object is to determine the proportions of the substances so that the composition of the blend is within specified tolerances or constraints, while minimizing the per unit cost of the blend.

The high speed capabilities of the analog computor offer the possibility of scanning, rather than *hill-climbing*, to achieve the desired solution in certain nonlinear programming problems where there may be more than one relative minimum of the cost function. An important practical example arises when the unknowns are constrained to have integral values. Succeeding issues of this journal will explore some of the interesting applications, if the Fates permit.

Appendix I

Development of Equations Suitable for Linear Algebraic Analog Computation

The n values are sought for the x_j's which minimize the sum of the m weighted squared errors arising in $m \geqslant n$ simultaneous equations of the form:

$$\sum_{j=1}^{n} a_{ij} x_j = b_i$$

The weighted errors are expressed by:

$$(1) \quad e_i = w_i \left(b_i - \sum_{j=1}^{n} a_{ij} x_j \right)$$

where w_i are the weighting factors, for which a value of unity often suffices. The sum of the weighted squared errors, namely

$$\sum e^2 = \sum_{i=1}^{m} \frac{1}{w_i} e_i^2 = \sum_{i=1}^{m} w_i \left(b_i - \sum_{j=1}^{n} a_{ij} x_j \right)^2$$

is minimized when:

$$\frac{\partial \sum e^2}{\partial x_j} = 2 \sum_{i=1}^{m} \frac{1}{w_i} \left(\frac{\partial e_i}{\partial x_j} \right) e_i = 0$$

But by differentiating Equation 1 there results:

$$\frac{\partial e_i}{\partial x_j} = -w_i a_{ij}$$

Employing the method of steepest descents, the desired solution is achieved as the steady state of the differential equation:

$$\frac{\partial \sum e^2}{\partial x_j} = -2T \frac{dx_j}{dt}$$

where T is an arbitrary dimensional constant controlling the solution time. Hence:

$$(2) \quad T \frac{dx_j}{dt} = \sum_{i=1}^{m} a_{ij} e_i$$

Equations 1 and 2 may be solved simultaneously with an analog structure. The solution will be stable provided that a unique solution exists. When steady state is achieved, the sum of the weighted squared errors is given by:

$$\begin{aligned} \sum e^2 &= \sum_{i=1}^{m} \frac{1}{w_i} e_i^2 \\ &= \sum_{i=1}^{m} \frac{e_i}{w_i} w_i \left(b_i - \sum_{j=1}^{n} a_{ij} x_j \right) \\ &= \sum_{i=1}^{m} b_i e_i - \sum_{j=1}^{n} \left(\sum_{i=1}^{m} a_{ij} e_i \right) x_j \end{aligned}$$

So that finally, by virtue of Equation 2, we obtain simply:

$$(3) \quad \sum e^2 = \sum_{i=1}^{m} b_i e_i$$

which enables evaluation of $\sum e^2$ by analog means.

Appendix II

Reduction to Symmetric Form

Alternatively, Equation 1 may be used to eliminate the e_i's from Equation 2 before using the analog equipment.

$$T\frac{dx_j}{dt} = \sum_{i=1}^{m} a_{ij}\, w_i \left(b_i - \sum_{k=1}^{n} a_{ik}\, x_k\right)$$
$$= \sum_{i=1}^{m} a_{ij}\, w_i\, b_i - \sum_{k=1}^{n}\left(\sum_{i=1}^{m} w_i\, a_{ij}\, a_{ik}\right) x_k$$

The subscripts j and k range from 1 to n corresponding to the number of unknowns,

Let:

$$b'_j = \sum_{i=1}^{m} w_i\, a_{ij}\, b_i$$
$$a'_{jk} = a'_{kj} = \sum_{i=1}^{m} w_i\, a_{ij}\, a_{ik}$$

Then

$$\text{(4a)} \quad T\frac{dx_j}{dt} = b'_j - \sum_{k=1}^{n} a'_{jk}\, x_k$$

But allowing T to become arbitrarily small:

$$\text{(4)} \quad x_j = \frac{1}{a'_{jj}}\left[b'_j - \sum_{k=1}^{n} a'_{jk}\, x_k\right], \; k \neq j$$

The coefficients, b'_k and a'_{jk}, must be precalculated and then equation 4 or 4a may be solved for x_j using the analog. In steady state (or for $T \approx 0$), from Equations 3 and 1:

$$\sum e^2 = \sum_{i=1}^{m} b_i\, e_i = \sum_{i=1}^{m} w_i\, b_i\left(b_i - \sum_{j=1}^{n} a_{ij}\, x_j\right)$$
$$= \sum_{i=1}^{m} w_i\, b_i^2 - \sum_{j=1}^{n} x_j\left(\sum_{i=1}^{m} w_i\, a_{ij}\, b_i\right)$$

$$\text{(5)} \quad \sum e^2 = -\left(b'_0 + \sum_{j=1}^{n} b'_j\, x_j\right)$$

where:

$$b'_0 = -\sum_{i=1}^{m} w_i\, b_i^2$$

If Equation 4 (or 4a) is used to solve an arbitrary symmetric matrix the formulation may not be stable. All diagonal elements, a'_{jj}, must be positive and should be larger than the other elements in the same row or column, in order to avoid instability.

A stable symmetric matrix equation often results from the mathematical description of discrete linear resistive, inductive, and capacitive fields or of their equivalents in nonelectric media. For example consider a mechanical field in which the applied forces f_j are related to deflections q_k by stiffnesses k_{jk}.

$$f_j = \sum_{k=1}^{n} k_{jk}\, q_k, \; (k_{jk} = k_{kj})$$

The algebraic analog machine may be used to find the values of the deflections q_k (Equation 4), given f_j and k_{jk}, and in addition to determine the stored energy E (Equation 5 with $b'_0 = 0$).

$$E = \frac{1}{2}\sum_{j=1}^{n} f_j\, q_j$$

Furthermore, the same techniques can be applied with slight modification when the coefficients and unknowns are complex numbers. This type of equation frequently results from the design analysis of carrier or AC amplification and transmission systems.

Appendix III

Linear Programming

The n values of x_j are sought which maximize the linear profit function P, the negative of the "cost" function referred to in the text.

$$\text{(6)} \quad P = \sum_{j=1}^{n} c_j\, x_j$$

subject to m linear constraints:

$$b_i - \sum_{j=1}^{n} a_{ij}\, x_j \leqslant 0$$

Quadratic terms (having a negative definite coefficient matrix) in the profit function may be treated as a set of linear equality constraints the sum of whose weighted squared errors must be minimized.

When no constraints are violated, the method of steepest ascents is used to cause P to increase with time

$$T\frac{dx_j}{dt} = K\frac{\partial P}{\partial x_j} = Kc_j$$

until a constraint boundary is reached. Then we wish to follow the boundary, choosing the direction which increases P, until the maximum results. When this occurs, n of m constraints are satisfied as equalities.

The weighted errors of the inequality constraints are defined as:

$$\text{(7)} \quad e_i = \mathbf{U}\left\{0, w_i\left(b_i - \sum_{j=1}^{n} a_{ij}\, x_j\right)\right\}$$

Where U is the upper selection operator, or least upper bound, implying that e_i is the larger of the two arguments.

Any of the x_j's may be constrained to be non-negative without employing an e_i equation by adding a term to Equation 8. An equality constraint is achieved by omitting the upper selection operation.

The largest profit function P, that will produce a minimum (usually zero) in $\sum e^2$ is, achieved when:

$$\text{(8)} \quad T\frac{dx_j}{dt} = Kc_j + \sum_{i=1}^{m} a_{ij}\, e_i$$

provided K is sufficiently small. The sum of weighted squared constraint errors:

$$\sum e^2 = \sum_{i=1}^{m} \frac{1}{w_i} e_i^2$$

in steady state is given by:

$$\text{(9)} \quad \sum e^2 = \sum_{i=1}^{m} b_i\, e_i + KP$$

Equations 6, 7, 8, and 9 may be solved by an analog. In addition the special purpose analog will identify those constraints which are inactive and hence irrelevant. Inaccuracy caused by a finite K can be eliminated by setting K to zero, removing all irrelevant constraints, and converting all inequality constraints to equalities, thus achieving a solution of n constraint equations for n unknowns.

Amplifiers: continued from page 2

current load) are away from ground, and a few of the most popular do-it-yourself boosters are shown.

Figure 3a shows a circuit having current gain, in which the source needs supply only R_0/R_1 of the load current. Three typical booster arrangements are shown in Figures 3b and 3c, and an unusual *shunt* booster circuit is shown in Figure 3d. The shunt booster circuit is useful where high voltages would lead to large amounts of dissipation and undue voltage stress on circuit elements; but note that the amplifier polarity must be reversed, and that some care may be required to attain dynamic stability.

DIFFERENTIAL-INPUT AMPLIFIERS
(and single-ended equivalents using amplifier pairs)

It is possible to construct a quite simple current source, using a differential amplifier, a floating reference source which need supply no current, and an adjustable resistor. This current source will, within the ratings of the amplifier, supply current to a load whose "other terminal" can be "anywhere," if there is a composite return path for current to amplifier common, and if the closed-loop transfer characteristic is stable. The load is commonly represented as a single element connected to common, for simplicity. In Figure 4, the follower-connected output is equal to the voltage at the positive input, which is equal to the load voltage plus the battery voltage. Hence, the voltage across the resistor R_0 must be maintained equal to the battery voltage, and the current through the load must therefore be E_0/R_0, in the absence of error. In all circuits discussed here, the possible need for capacitance across feedback resistors, and (to a judicious degree) across the load, to preserve dynamic stability, is implicit. In addition to the usual current and voltage offset errors common mode error should also be considered and minimized, or compensated for.

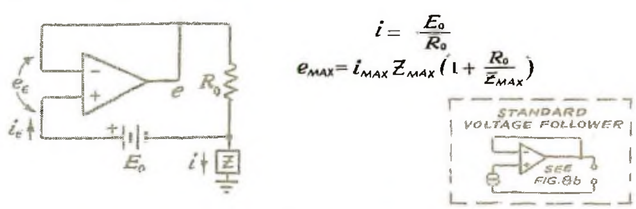


Fig. 4 Voltage to Current Transconductor (Floating source, grounded load, differential amplifier; source supplies no signal current)

The circuit of Figure 4 may be considered a degenerate case of the circuit of Figure 5a, which has fascinating properties. An equivalent circuit, which could employ two single-ended (conveniently chopper-stabilized) amplifiers, is shown in Figure 5b. This circuit, first described to us by Lincoln Laboratory, is known in our organization as the "Howland" Circuit. It can provide a current "to ground" proportional to the sum and difference of any number of grounded source voltages. Only one pair of such voltages is shown in Figure 5. Note that in the circuit of Figure 5a, the direct input supplies full current at short circuit, no current when $Z_L = R_0$, and negative current when Z_L is greater than R_0. The inverting input supplies i/K at short circuit, and increasing current as Z_L increases.

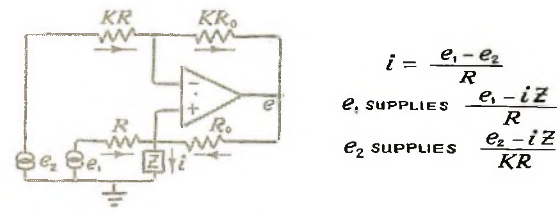


Fig. 5a Differential Voltage to Current Transducer (Grounded sources, grounded load, differential amplifier)

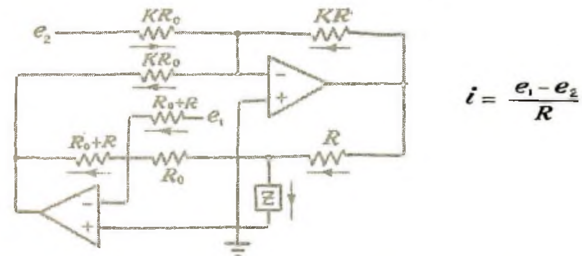


Fig. 5b Differential Voltage to Current Transducer (Grounded sources, grounded load, two single-ended amplifiers)

NEGATIVE IMPEDANCE

By definition, an ideal current source will have infinite impedance. One can easily satisfy oneself that all the current sources described above come quite close to meeting this requirement. Simply let the load impedance approach open circuit conditions, and observe that the load voltage strives toward saturation, even for quite small currents. But there are other interesting possibilities suggested by the circuit of Figure 5a (and its two-amplifier equivalent* Figure 5b).

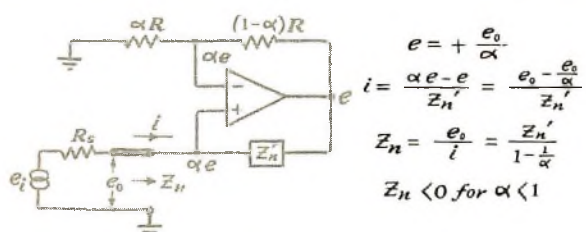


Fig. 6a Negative Impedance Circuit

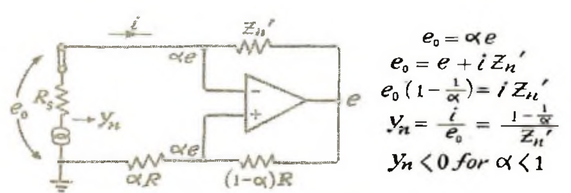


Fig. 6b Negative Admittance Circuit

These will be seen in Figure 6, in which are shown two varieties of negative impedance: in 6a the "short circuit stable* negative impedance" (or, more simply, *negative impedance*); and in 6b the "open circuit stable* negative impedance" (or, more simply, *negative admittance*). In both these circuits, it is possible to have negative resistance, capacitance, inductance, or com-

* Karplus, W. J., *Analog Simulation*, McGraw-Hill, 1958, pp. 257–259.

binations thereof. The circuit of Figure 6a will be unconditionally stable (assuming ideal amplifier and circuitry) if the magnitude of the source impedance in parallel with any shunt impedance to ground is always less than Z_n, and stable under some conditions if this magnitude is less but proper phase relations are maintained.

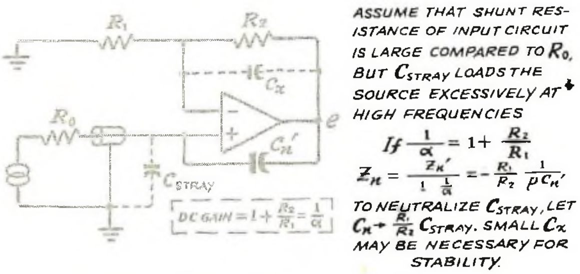


Fig. 6c Neutralizing Input Capacitance

One possibly useful and suggestive application is that of an active network having negative capacitance to compensate for lead capacitance in the amplifier input and associated circuitry. The elements of such a circuit are shown in Figure 6c. Although tailoring the dynamics of the amplifier may be necessary to insure stability, the neutralization of capacitance at the input can make possible high impedance measurements that would otherwise be all but unattainable. There are obviously many other applications for an accurate and flexible negative driving-point impedance or admittance.

It is interesting to note that the circuits of Figs. 6a and 6b resemble the Howland circuit with infinite impedance load, when the positive source impedances are equal to Z_n (i.e., at the point of instability).

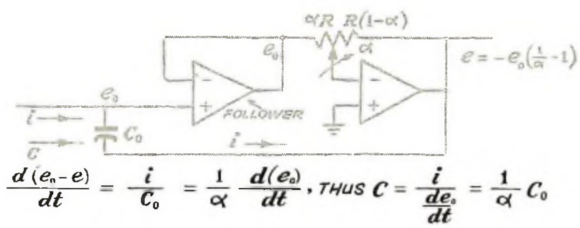


Fig. 7a Capacitance-Stretcher Circuit

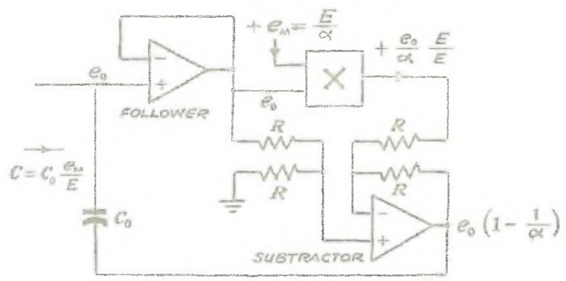


Fig. 7b Modulated Capacitance

MISCELLANEOUS TOPICS

1. This business of impedance transformation is not limited to positive to negative transformations alone. Consider the circuit of Figure 7a, in which we achieve a result opposed to that of Figure 6c, an *increase* in the value of a capacitance. Using this basic arrangement, we can linearly modulate the size of a capacitance with a voltage, as shown in Figure 7b, using an analog multiplier, or a servoed potentiometer. Because the amplifier has gain, circuit voltages must be such as not to drive the amplifier output into saturation.

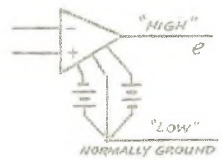


Fig. 8a
Operational Amplifier, Showing Differential Inputs and Outputs

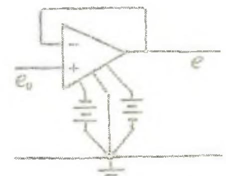


Fig. 8b
Conventional Follower

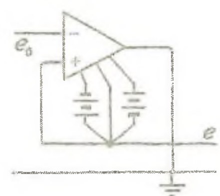


Fig. 9a
Inverted Follower (Could utilize chopper-stabilized amplifier)

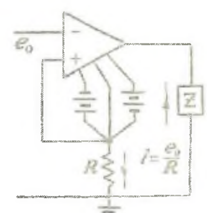


Fig. 9b
Inverting Amplifier as Voltage to Current Transconductor

2. A new dimension of flexibility can be gained (and lost) by allowing the power supply to float. (Separate power supplies are then needed for each amplifier.) The differential operational amplifier has available a negative and a positive input, and a positive and a negative output (See Figure 8). If either output terminal can be grounded, as well as either input terminal, new possibilities for measurement become available, particularly for chopper-stabilized amplifiers, in which the "plus" input and "minus" output are normally tied together irrevocably. For example, a stabilized follower having extremely high input impedance is practicable (Figure 9a). And in Figure 9b a current generator is shown, having grounded source and grounded load. In this instance, the grounded source draws no current: It is the inverted version of Figure 4. Verification of this relationship is left to the Reader, assuming the Reader has read to this point with sufficient admittance (not negative).

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