LOW-COST TRIANGLE-INTEGRATION MULTIPLIERS FOR ANALOG COMPUTERS

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SUMMARY.

A new simple electronic multiplier achieves a static accuracy of 0.1 percent of full scale at extraordinarily low cost : only two d-c amplifiers sharing a common chopper are required, and a single two-tube triangle generator suffice for several multpliers. A new precision limiter circuit developed for the multiplier has other applications as well.

1. *Triangle-Integration Multipliers.*

Figure 1 illustrates the operation of a simple triangle-integration multiplier designed to produce an output voltage proportional to the product XY of two input voltages X and Y.

In the upper limiter block of Fig. 1a, an accurately-shaped triangular waveform $V(t)$ measuring $2\,V_0$ volts peak-to-peak is added to the inverted bias voltage $X - Y$; the resulting waveform is then half-wave rectified or clipped at zero voltage (Fig. 1b). The

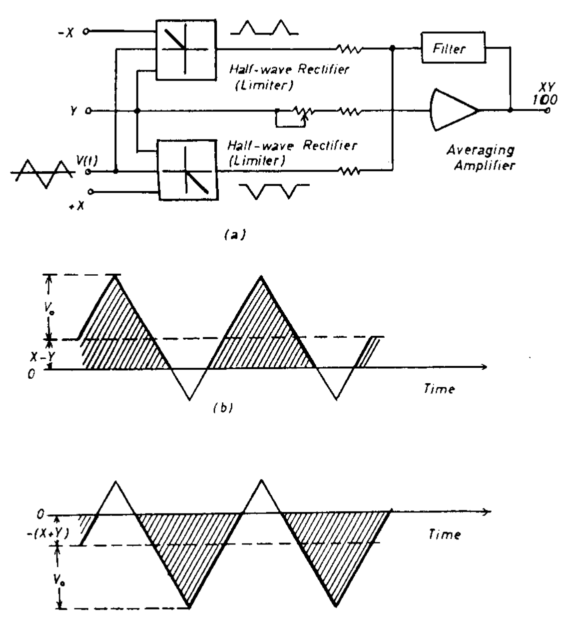

Fig. 1. — Block diagram (a) and waveforms at limiter output terminals (b, c) for triangle-integration multiplier n° 1.

average value of the rectified output is proportional to the area of each shaded triangle and equals

$$E_{AV} = \frac{1}{4\,V_0}\,(V_0 + X - Y)^2$$

$$= \frac{V_0^2 + X^2 + Y^2}{4\,V_0} + \frac{X - Y}{2} - \frac{XY}{2\,V_0} \qquad (1)$$

In the lower limiter block of Fig. 1a, similar biasing and *negative* half-wave rectification (Fig. 1c) yields clipped triangles of opposite polarity and average value

$$E_{AV} = -\,\frac{1}{4\,V_0}\,(V_0 + X + Y)^2$$

$$= -\,\frac{V_0^2 + X^2 + Y^2}{4\,V_0} - \frac{X + Y}{2} - \frac{XY}{2\,V_0} \qquad (2)$$

The averaging amplifier in Fig. 1a is a low-pass summing amplifier. It inverts and adds the average values (1) and (2) together with the input voltage Y to produce the desired multiplier output voltage

$$-b\,\left(-\frac{XY}{2\,V_0} - \frac{XY}{2\,V_0} - \frac{Y}{2} - \frac{Y}{2} + Y\right)$$

$$= \frac{b}{V_0}\,XY = \frac{XY}{100} \qquad (3)$$

where X and Y can be positive, negative, or zero (four-quadrant multiplication), as long as $|X| + |Y| \leqslant V_0$.

Several basically similar schemes for triangle-integration multiplication exist [1-5]. Thus the alternative circuit illustrated by the block diagram and waveforms of Fig. 2 employs two biased limiters (amplitude-selection circuits or dead-space circuit) [5] set at $\pm(X + A)$ to obtain the desired product in the form

$$\frac{-b}{4\,V_0}\,[(V_0 - Y - X - A)^2 - (V_0 + Y - X - A)^2$$

$$+ 2\,(V_0 - A)\,Y] = \frac{XY}{100} \qquad (4)$$

where $X + Y + A \leqslant V_0$; the fixed bias voltage A is used if the two limiters are to operate on either side of zero (dead-space circuits).

Figure 3 shows yet another triangle-integration multiplier, this time based on the use of a dual-limiter circuit. Like servo multipliers, the multipliers shown in Figs. 1 to 3 require both $+X$ and $-X$ inputs; the required phase inverter is often already included

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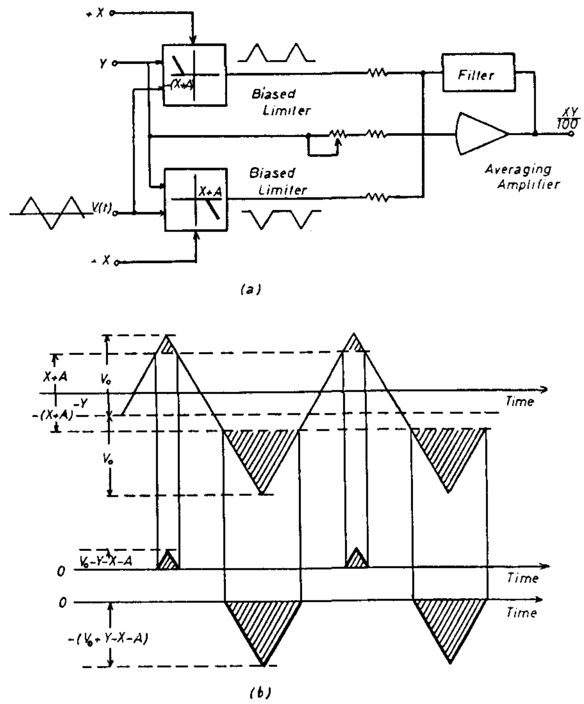


Fig. 2. — Block diagram (a) and waveforms (b) for triangle-integration multiplier n° 2.

in the computer setup. One can conveniently invert the sign of the multiplier output by interchanging the + X and — X inputs.

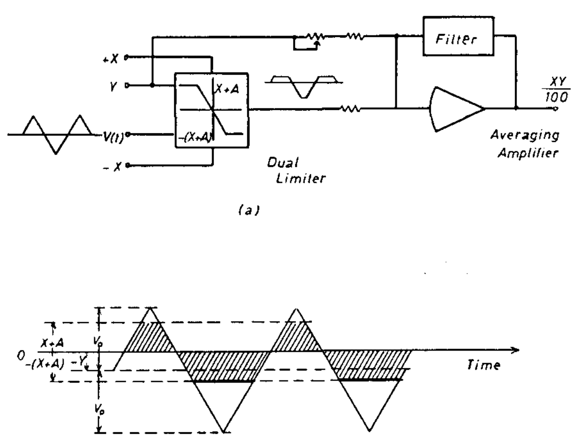


Fig. 3. — A third type of triangle-integration multiplier.

2. *Design Considerations.*

The dynamic accuracy (frequency response) of a triangle-integration multiplier depends on the « carrier » frequency of the triangular waveform (1 KC to 1 MC); with a well-designed averaging filter phase-shift errors below 10 deg may be expected at 1/100 of the carrier frequency. The static (d-c) accuracy depends on the accuracy of the triangle generator and on that of the limiter circuits, although some errors due to these components tend to cancel in the subtraction or in push-pull circuits. Since the performance of amplifiers and limiter diodes deteriorates with increasing carrier frequency, one must make a compromise between static and dynamic accuracy, as in all modulation-type multipliers.

Existing triangle generators are either relatively inaccurate or somewhat complex [1-5]. The better circuits employ as many as six chopped-stabilized d-c amplifiers to cancel errors by computing products in the symmetrical form 1/2 (XY + YX) and to eliminate the effect of variations in the triangle amplitude V_0 by combining the multiplier with a division circuit, which is also useful in its own right.

The two new multipliers described here employ precision limiter circuits together with a moderately low carrier frequency (5 and 10 KC) in an attempt to obtain respectable accuracy with a minimum of components.

3. *A practical Circuit : Multiplier No. 1.*

A *precision limiter circuit* [6] compared in Fig. 4 with an ordinary series-diode limiter, divides distortion and forward impedance of the limiting diode D_1 by the loop gain of a high-gain feedback amplifier. Moreover, as the limiting diode cuts off, the high-gain d-c amplifier loses its degenerative feedback and cuts the diode off sharply, so that a very accurate break-point characteristic results. When D_1 is off, D_2 is on, so that feedback keeps the summing point, and hence the limiter output, at zero voltage. Reversal of diodes and bias voltages yields a similar limiter with positive output.

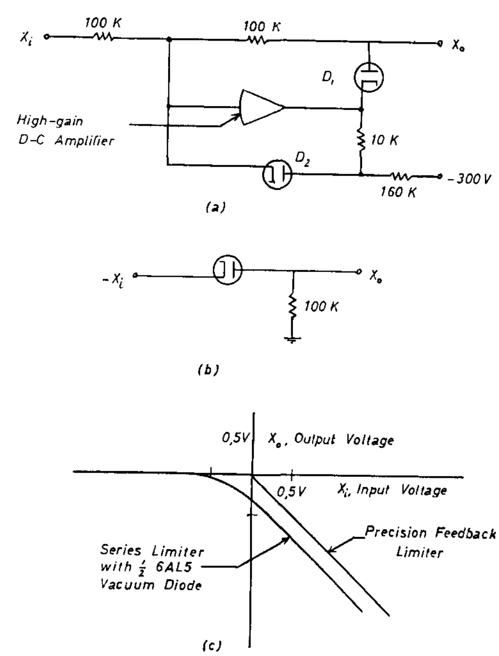
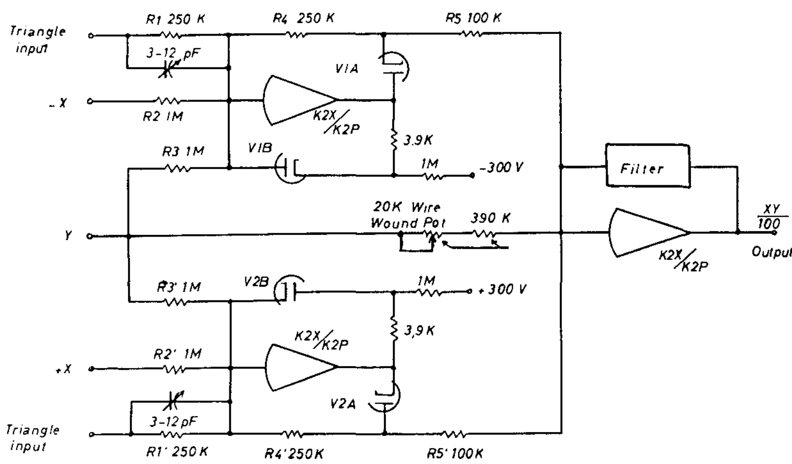


Fig. 4. — Precision limiter (a), simple series limiter (b), and comparison of actual transfer characteristics at 0.1 CPS (c). Static (D-C) accuracy is within 0 : 0.5 volt.

The triangle-integration multiplier circuit of Fig. 5 implements the block diagram of Fig. 1 with precision limiters. The circuit is designed to work with V_0 = 60 volts; to obtain input and output ranges of ±100 volts, the feedback resistors effectively reduce

X and Y by a factor of 1/4 at the limiter output, so that $V_0 = 60 \geqslant 1/4\ (|X| + |Y|)$, and the limiter output voltage is within ± 110 volts. The output amplifier rescales the output to equal XY/100 volts.

With a carrier frequency of 5 KC, this multiplier yielded a static accuracy better than 0.05 percent of full scale (200 volts), using three low-cost chopper-stabilized Philbrick K2-X/K2-P d-c amplifiers. Although multiplier No. 1 would operate satisfactorily with a 10 KC carrier, all further work was done with multiplier No. 2, which requires only 2 d-c amplifiers.



All resistors are ± 1 % wire wound types.
Pairs: (R1, R1'), (R2, R2'), (R3, R3'), (R4, R4') and (R5, R5') are matched to within ± 0.1 % of each other.
V1, V2 are 6 AL 5 vacuum diodes.

Fig. 5. — Practical circuit for multiplier N° 1.

4. *An Accurate Multiplier at Minimum Cost.*

In an attempt to replace the two limiter amplifiers used in multiplier No. 1 by a single one, the block diagram of Fig. 3 was tried first with a new precision dual limiter employing a diode-bridge feedback circuit [7]. Since the dual limiter, although accurate within 0.1 volt at d-c, did not operate with the required accuracy at 5 KC, it was successfully replaced by the new *precision dead-space limiter* of Fig. 6 in the block-diagram arrangement of Fig. 2.

In the new circuit of Fig. 6a [8], unity feedback through the diode bridge $D_1\, D_2\, D_3\, D_4$ keeps the output of amplifier 1 at zero for $-RE_1/r_1 < X < RE_2/r_2$. If X exceeds these limits the bridge is cut off sharply by the high amplifier gain, and either D_5 or D_6 is turned on sharply, as in any precision limiter. The limiter output voltages X_1, X_2 are either accurately equal to zero, or they are feedback-compensated for distortion and internal impedance. The static (d-c) accuracy is within 0.05 volts, provided that the resistances r_1 and r_2 exceed the diode forward resistance by a factor of at least 2000. Figure 6b demonstrates accurate limiting at 10 KC.

Figure 7 shows a triangle-integration multiplier using the new dead-space limiter in the circuit arrangement of Fig. 2. The resistance networks are scaled so that X and Y are reduced by respective factors of 1/8 and 1/4 at the limiter output, with $V_0 = 60$ volts. Fixed bias voltages $\pm A$, derived from the computer reference supply, are added to $\pm X$ (see also Fig. 2), so that the limiter breakpoints vary between ± 10 volts ($X = -100$ volts) and ± 35 volts ($X = +100$ volts); the breakpoint voltages cannot exceed $|V_0| - 1/4\ |Y|$ in absolute

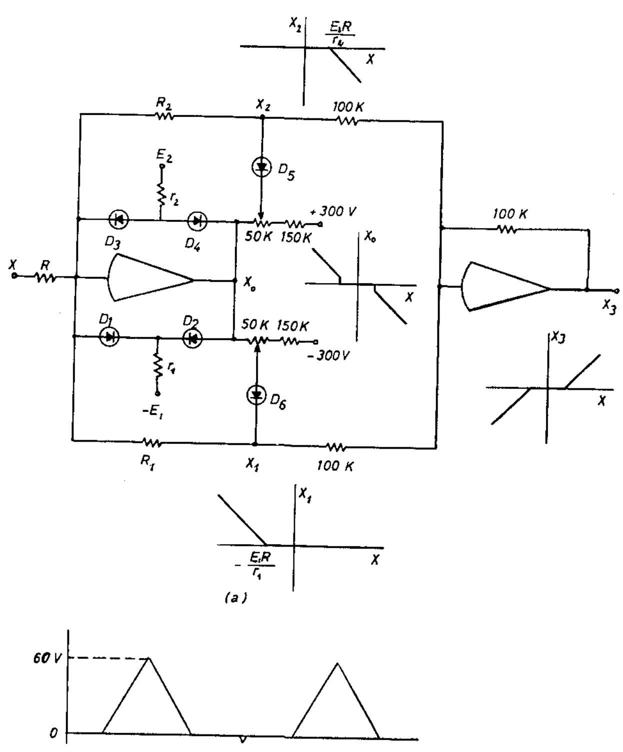
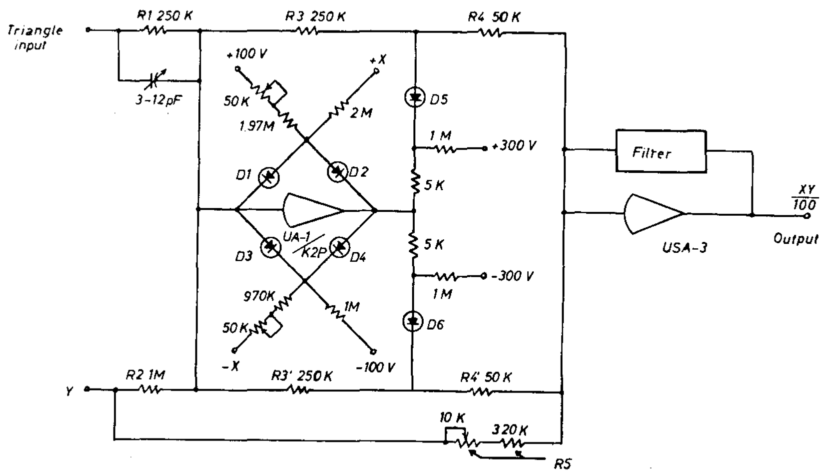


Fig. 6. — New precision dead-space limiter (a) and clipped 10 KC triangular-wave voltage (b).

The pot associated with R5 is wire wound. The others are carbon.
All resistors are ± 1 % wire wound types.
(R3, R3') and (R4, R4') are matched to within ± 0.1 % of each other.
All diodes are IN 643 silicon fost recovery.



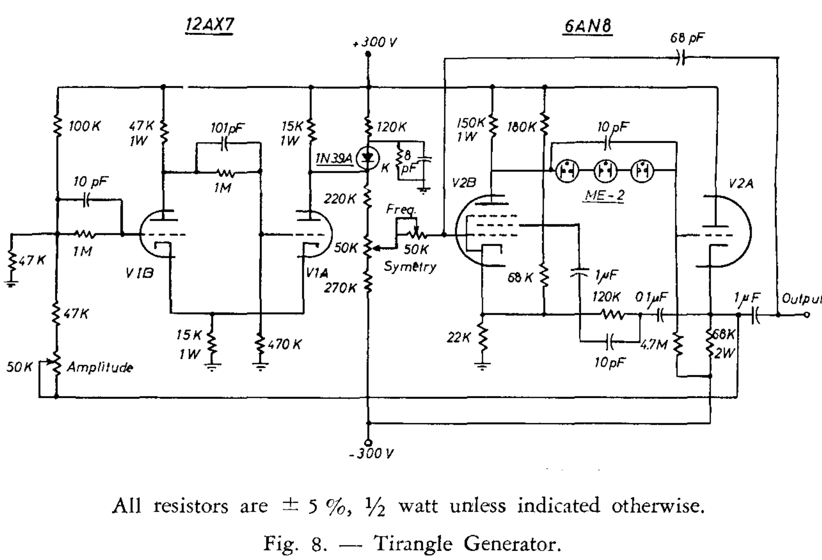
The pot associated with R5 is wire wound. The others are carbon.
All resistors are ± 1 % wire wound types.
(R3, R3') and (R4, R4') are matched to within ± 0.1 % of each other.
All diodes are IN 643 silicon fost recovery.

Fig. 7. — Practical circuit for multiplier N°2.

value. The output amplifier rescales the output to XY/100 volts.

5. *Triangle-Generator and Filter Circuits.*

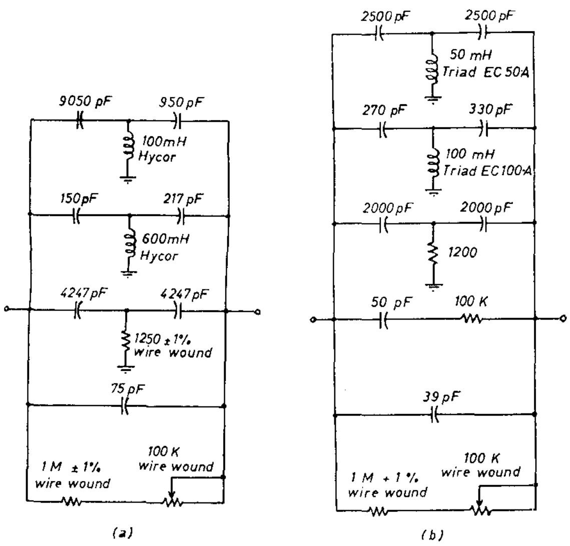
The triangle-generator circuit of Fig. 8 generates a symmetrical triangular waveform by means of a feedback loop comprising a diode-limited Schmitt trigger circuit (V1) and a Miller integrator (V2) [7]. The integrator output is capacitance-coupled to both the integrating capacitor and the output terminal, so that no chopper stabilization is needed. A slight amount of internal regeneration raises the integrator forward gain sufficiently to produce the required 0.1 percent integration linearity. The circuit permits fine adjustment of the triangle frequency to match the ripple-filter notch, and of the triangle amplitude V_0. With the regulated ±300 volts computer power supplies V_0 does not usually require adjustment.



All resistors are ± 5 %, ½ watt unless indicated otherwise.

Fig. 8. — Tirangle Generator.

Figure 9 shows the ripple-filter networks used with multiplier No. 1 at 5 KC and with multiplier No. 2 at 10 KC. The filters have notches at the second harmonic as well as at the fundamental triangle frequency. Some care is needed to avoid stray coupling of the carrier waveform to the output-amplifier summing point.



All components are ± 5 % unless otherwise noted.

Fig. 9. — Filter networks used with multiplier N° 1 (a) and with multiplier N° 2 (b).

6. *Results and Discussion.*

The circuit of Fig. 7 was built using 1 N 643 diodes and 0.1 percent wire wound resistors in all critical networks. Table 1 summarizes the multiplier specifications, with a Philbrick USA-3 output amplifier, and a UA-Mod 1 limiter amplifier. The UA-Mod 1 [9] is a faster and more powerful plug-in amplifier than the Philbrick K2-X; the latter can be substituted with similarly favorable results if all computing voltages are restricted between —50 volts and +50 volts.

Considering its accuracy, the new multiplier is extraordinarily low in cost, since the only *electronics required are two d-c amplifiers and six diodes, plus a two-tube triangle generator shared between 4 or 5 multipliers.* The two d-c amplifiers could mount on a common printed-circuit card bearing only 6 vacuum-tube envelopes and a common chopper for both amplifiers. Four such cards could be combined with a common triangle-generator card to provide four respectably accurate multiplier channels at a retail cost of about $200.00 per channel, less power supplies.

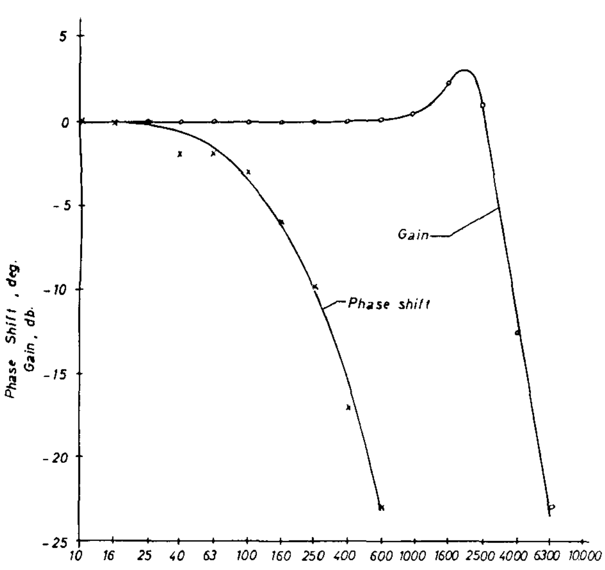


Fig. 10. — Amplitude response and phase response of multiplier N° 2.

At this price the multiplier should compare favorably with medium-cost time-division multipliers [5] of similar performance. Diode quarter-square multipliers [5] of comparable cost will be less accurate but have better frequency response.

With suitable changes in the output-amplifier feedback circuit, the multiplier of Fig. 7 should be adaptable to the usual *division circuits* [5] using implicit computation, and to low-cost « *generalized* » *integration* with respect to a voltage Z, based on the relation

$$\int_0^t X\,dZ = \int_0^t X\,\frac{dZ}{dt}\,dt$$

with the output amplifier functioning as the integrator.

7. *Limiter-Circuit Applications.*

The precision dead-space limiter circuit of Fig. 6a was developed specifically for the multiplier [8], but is of interest in its own right. Figure 11 shows two useful applications.

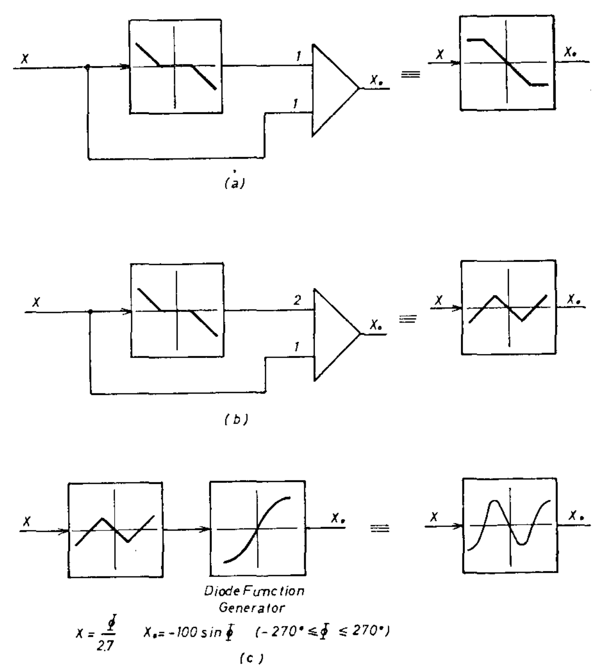


Fig. 11. — Use of the precision dead-space circuit in a dual limiter (a) and in a precision quadrant switch (b) for an electronic sine generator (c).

8. *Acknowledgements.*

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TABLE 1 :

SPECIFICATIONS FOR MULTIPLIER NO. 2

(a) (Carrier Frequency : 5 KC - V_0 = 60 volts).
Static Accuracy : 0.08 percent of full scale (200 volt), all four quadrants.
Accuracy with 200 volt peak-to-peak 1 cps triangular waveform on one input and ±100 volt on the other : 0.1 percent.
Same with 10 cps triangular waveform : 0.7 percent.
Frequency response for full-scale output (200 volt peak-to-peak sinusoidal voltage multiplied by 100 volt) either input :
3 db point : 1.2 Kc
5 deg. phase shift point : 60 cps.
Output Ripple, max. amplitude : 1.2 volt.

(b) (Carrier Frequency : 10 KC - V_0 = 60 volts).
Static Accuracy : 0.08 percent of full scale (200 volt), all four quadrants.
Accuracy with 200 volt peak-to-peak 1 cps triangular waveform on one input and ±100 volt on the other : 0.1 percent.
Same with 10 cps triangular waveform : 0.7 percent.
Frequency response for full-scale output (200 volt peak-to-peak sinusoidal voltage multiplied by 100 volt) either input :
3 db point : 3 Kc
5 deg. phase shift point : 130 cps.
Output Ripple, max. amplitude : 0.7 volt.

REFERENCES

[1] Meyer, R.A. and H.B. Davis, Triangular-wave Analog Multiplier, *Electronics,* Aug. 1956.

[2] Pfeiffer, P.E., A Four-Quadrant Multiplier, *National Simulation Conference,* Dallas, 1958.

[3] General Electric (Phoenix), *Advance Bulletin on GE Analog Modules,* 1959.

[4] G.A. Philbrick Researches, Boston, Massachusetts, *Specification Sheets for MU/DV and K5-M Multipliers.*

[5] Korn, G.A., and T.M. Korn, *Electronic Analog Computers,* 2nd Ed., McGraw-Hill, N.Y., 1956.

[6] Morrill, C.D. and R.V. Baum, Diode Limiters Simulate Mechanical Phenomena, *Electronics,* November, 1952.

[7] Koerner, H. and G.A. Korn, Function Generation with Operational Amplifier, *Electronics,* November 6, 1959.

[8] Korn, G. A., A Preciscion Dead-space Limiter for Triangle-Integration Multipliers, *ACL Memorandum No. 14,* November 25, 1959, Electrical Engineering Department, The University of Arizona.

[9] Koerner, H., Plug-in Operational Amplifiers with Extended High-frequency Response, *ACL Memorandum No. 13,* February 17, 1960, Electrical Engineering Department, The University of Arizona.