PHASE DISTORTION IN OPERATIONAL AMPLIFIERS

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ABSTRACT

The stray capacities in the input and feedback paths of an operational amplifier, its finite bandwidth and d.c. gain effectively narrow the bandwidth of an amplifier causing amplitude and phase errors in the computer solutions. A given degree of accuracy consequently becomes more difficult to obtain with an amplifier when used in a high-speed repetitive computer than when used in a slower machine. In a high-speed computer, phase distortion is more serious than the amplitude distortion.

An attempt has been made in this paper to analyze an operational amplifier with its associated input-feedback networks and a practical method has been suggested for reducing the undesired phase shift.

Introduction.

Macnee [1] has shown how the presence of undesired time constants associated with operational amplifier networks effectively narrow the bandwidth of an operational amplifier and consequently introduce amplitude and phase errors in the computer solutions of the differential equations. The errors get larger, and hence a given degree of accuracy becomes more difficult to obtain with an amplifier when used in a high-speed repetitive computer than when used in a slower machine.

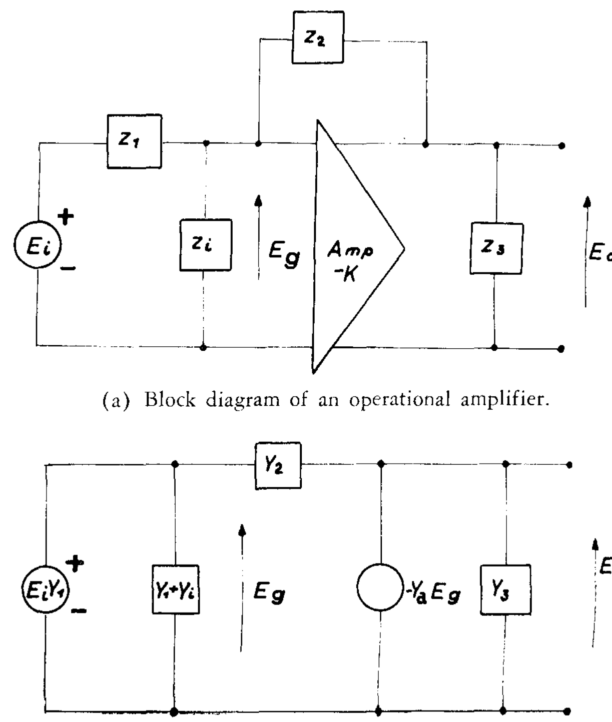
Bell and Rideout [2] in their analysis of an operational amplifier have assumed an amplifier with a finite bandwidth and the effect of finite bandwidth has been represented by a single equivalent time constant. The authors, however, have neglected the presence of stray capacities that are invariably associated with input and feedback networks of the amplifier. The authors have pointed out and emphasised the need for guarding against phase errors which are more serious than amplitude errors in the repetitive type of computers.

An attempt has been made in this paper to analyse a physically realizable operational amplifier along with its input and feedback networks. The effect of finite bandwidth of an amplifier is assumed to be represented by a single equivalent time constant and the stray capacities in the input and feedback networks are taken into account.

Analysis of an Operational Amplifier.

Simplified block diagramatic representation of an operational amplifier is shown in figure 1 (a), where

Z_1 = series input impedance
Z_i = amplifier input impedance
Z_2 = feedback impedance
Z_3 = output impedance
$K = G_a R_3$ = d.c. gain of the operational amplifier
$Y_a = G_a/(1 + s\,T_a)$
T_a = amplifier time-constant.



(a) Block diagram of an operational amplifier.

(b) Equivalent circuit of an operational amplifier.

Fig. 1. — Block diagram of an operational amplifier and its equivalent circuit.

An equivalent circuit of figure 1 (a) is shown in figure 1 (b). The nodal equations by inspection may be written as:

$$E_i\,Y_1 = (Y_1 + Y_i + Y_2)\,E_g - Y_2\,E_0 \qquad (1)$$

$$0 = -(Y_2 - Y_a)\,E_g + (Y_2 + Y_3)\,E_0 \qquad (2)$$

Solving equations (1) and (2) for E_0

$$\frac{E_0}{E_i} = -\frac{Y_1\,(Y_a - Y_2)}{Y_2\,(Y_1 + Y_i + Y_3 + Y_a) + Y_3\,(Y_1 + Y_i)} \qquad (3)$$

Equation (3) is a general expression for the transfer function of an operational amplifier and its associated

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input feedback networks. If the amplifier had infinite gain very wide bandwidth, infinite input impedance and very low output impedance, then equation (3) will reduce to the well known form:

$$\frac{E_0}{E_i} \cong -\frac{Y_1}{Y_2}.$$

In a high-speed repetitive type of computer phase distortion is more serious than amplitude distortion as it causes large phase shifts with fewer amplifiers resulting in instabilities in the computer loop.

It would be of interest to derive from equation (3) an expression for the phase distortion for the case of a scaler unit. In a scaler Z_1 and Z_2 are usually resistive. But due to the presence of stray capacities at the input and in the feedback path of the amplifier the reactive components are also present. In a well designed operational amplifier intended for precision analogue computers the output stage is usually a cathode follower with a very low output impedance and hence it would be reasonable to assume the output impedance Z_3 small and purely resistive ($Z_3 = R_3$).

Further if,

$$Y_1 = \frac{1}{Z_1} = \frac{(1 + s\,T_1)}{R_1} \tag{4}$$

$$Y_i = \frac{1}{Z_i} = \frac{(1 + s\,T_i)}{R_i} \tag{5}$$

$$Y_2 = \frac{1}{Z_2} = \frac{(1 + s\,T_2)}{R_2} \tag{6}$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{R_3} \tag{7}$$

$$Y_a = \frac{G_a}{(1 + s\,T_a)} = \frac{K}{R_3\,(1 + s\,T_a)} \tag{8}$$

$$K = G_a\,R_3 \tag{9}$$

Then substituting equation (4) through (9) in equation (3) and simplifying *

$$\frac{E_0}{E_i} = -R_i\,\frac{(A + B\,s)}{(C + D\,s)} \tag{10}$$

where

$$A = (K\,R_2 - R_3) - \{R_3\,T_2\,T_a + R_3\,T_1\,(T_a + T_2)\}\,s^2 \tag{11}$$

$$B = \{T_1\,(K\,R_2 - R_3) - R_3\,(T_a + T_2)\} - R_3\,T_1\,T_2\,T_a\,s^2 \tag{12}$$

$$\begin{aligned} C = {} & [R_i\,R_3 + R_1\,R_3 + R_1\,R_i + K\,R_1\,R_i + R_2\,(R_1 + R_i)] \\ & + [R_3\,T_a\,(R_i\,T_1 + R_1\,T_i) + T_2\,\{R_i\,R_3\,(T_1 + T_a) \\ & + R_1 R_3\,(T_a + T_i) + R_1\,R_i\,T_a\} \\ & + R_2\,T_a\,(R_i\,T_1 + R_1\,T_i)]\,s^2 \end{aligned} \tag{13}$$

* Details are shown in the Appendix.

$$\begin{aligned} D = {} & R_i\,R_3\,(T_1 + T_a) + R_1\,R_3\,(T_a + T_i) \\ & + R_1\,R_i\,T_a + T_2\,\{R_i\,R_3 + R_1\,R_3 + R_1\,R_i + K\,R_1\,R_i\} \\ & + R_2\,\{R_i\,T_1 + R_1\,T_i + T_a\,(R_1 + R_i)\} \\ & + R_3\,T_2\,T_a\,(R_i\,T_1 + R_1\,T_i)\,s^2 \end{aligned} \tag{14}$$

The phase distortion from equation (10) is seen to be

$$\phi = \tan^{-1}\left(\frac{B\,\omega}{A}\right) - \tan^{-1}\left(\frac{D\,\omega}{C}\right) \tag{15}$$

and for small angles, equation (15) may be rewritten as

$$\phi \cong \left(\frac{B\,\omega}{A}\right) - \left(\frac{D\,\omega}{C}\right) \tag{16}$$

Since K is very large, R_1, R_2, R_i are large compared to R_3 and T_a is large compared to T_1, T_2, T_i and for values of ω in the range of interest ($f < 10$ Kc/sec). Equation (10) after necessary simplification reduces to:

$$\left.\frac{E_0}{E_i}\right|_{j\omega} \cong -\frac{R_2}{R_1}\,\frac{(1 + j\,\omega\,T_1)}{1 + j\,\omega\,\{T_2 + \frac{T_a}{K}(1 + \frac{R_2}{R_1} + \frac{R_2}{R_i})\}} \tag{17}$$

and consequently the phase distortion as given by equation (16) to:

$$\phi \cong \omega\,[T_1 - \{T_2 + \frac{T_a}{K}\,(1 + \frac{R_2}{R_1} + \frac{R_2}{R_i})\}] \tag{18}$$

It is evident from equation (18) that the phase distortion is small for values of ω in the range of interest if T_1, T_2, T_a very small, i.e. stray capacities small and wide bandwidth amplifier and K and R_i — the gain and input resistance of the amplifier — very large. A practical method for reducing the phase distortion is to make

$$T_1 = T_2 + \frac{T_a}{K}\,(1 + \frac{R_2}{R_1} + \frac{R_2}{R_i}) \tag{19}$$

by addition of a suitable capacitor in either the input or the feedback network so as to satisfy equation (19).

This method of phase correction has been successfully employed [3] in the design of WP Analogue Computer at the University of Wisconsin. The operational amplifier in the WP Computer consists of a plug-in-Philbrick $K_2\,x$ chopper-stabilized with a $K_2\,p$ amplifier with input and feedback resistors R_1 and R_2 each a 100 kilo-ohms. The phase distortion for an uncompensated scaler was measured to be about 0.5° at frequency as low as 2 Kc/sec while for a compensated scaler the phase distortion was unmeasurable (< 0.1°) for frequencies as high as 40 Kc/sec.

Conclusions.

The stray capacities that are invariably present in the input and feedback paths of an operational amplifier make greater contribution to phase errors than the finite bandwidth of the amplifier. A method for reducing these phase errors is to make the time constants of the input and feedback networks approximately equal.

Acknowledgement.

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APPENDIX

Block diagrammatic representation of an operational amplifier and its associated input feedback networks is shown in figure 1 (a) and its equivalent circuit is shown in fig. 1 (b). By inspection the nodal equations may be written as:-

$$E_i Y_1 = (Y_1 + Y_i + Y_2) E_g - Y_2 E_0 \qquad (20)$$

$$0 = -(Y_2 - Y_a) E_g + (Y_2 + Y_3) E_0 \qquad (21)$$

Solving equations (1) and (2) for E

$$\frac{E_0}{E_i} = -\frac{Y_1 (Y_a - Y_2)}{Y_2 (Y_1 + Y_i + Y_3 + Y_a) + Y_3 (Y_1 + Y_i)} \qquad (22)$$

let $\alpha = Y_1 (Y_a - Y_2)$ (23)

and $\beta = Y_2 (Y_1 + Y_i + Y_3 + Y_a) + Y_3 (Y_1 + Y_i)$ (24)

then

$$\frac{E_0}{E_i} = -\frac{\alpha}{\beta} \qquad (25)$$

if

$$Y_1 = \frac{1}{Z_1} = \frac{(1 + s T_1)}{R_1} \qquad (26)$$

$$Y_i = \frac{1}{Z_i} = \frac{(1 + s T_i)}{R_i} \qquad (27)$$

$$Y_2 = \frac{1}{Z_2} = \frac{(1 + s T_2)}{R_2} \qquad (28)$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{R_3} \qquad (29)$$

$$Y_a = \frac{G_a}{(1 + s T_a)} = \frac{K}{R_3 (1 + s T_a)} \qquad (30)$$

$$K = G_a R_3 \qquad (31)$$

Then from equations (23), (26), (28) and (30)

$$\alpha = \frac{(1 + s T_1)}{R_1} \left[\frac{K}{R_3 (1 + s T_a)} - \frac{(1 + s T_2)}{R_2}\right]$$

$$\alpha = \frac{(1 + s T_1)}{R_1} \left[\frac{-R_3 T_2 T_a s^2 - R_3 (T_2 + T_a) s + K R_2 - R_3}{R_2 R_3 (1 + s T_a)}\right]$$

$$\therefore \alpha = \frac{-a s^3 - b s^2 - c s + d}{R_1 R_2 R_3 (1 + s T_a)} \qquad (32)$$

where

$$\left.\begin{aligned} a &= R_3 T_1 T_2 T_a \\ b &= R_3 [T_2 T_a + T_1 (T_2 + T_a)] \\ c &= R_3 (T_2 + T_a) - T_1 (K R_2 - R_3) \\ d &= K R_2 - R_3 \end{aligned}\right\} \qquad (33)$$

Now from equations (24) and (26) through (31)

$$\beta = \frac{(1 + s T_2)}{R_2} \left[\frac{(1 + s T_1)}{R_1} + \frac{(1 + s T_i)}{R_i} + \frac{1}{R_3} + \frac{K}{R_3 (1 + s T_a)}\right] + \frac{1}{R_3} \left[\frac{(1 + s T_1)}{R_1} + \frac{(1 + s T_i)}{R_i}\right]$$

$$= \frac{(1 + s T_2)}{R_2} \left[\frac{R_i R_3 \{T_1 T_a s^2 + (T_1 + T_a) s + 1\} + R_1 R_3 \{T_i T_a s^2 + (T_i + T_a) s + 1\} + R_1 R_i (1 + s T_a) + K R_1 R_i}{R_1 R_i R_3 (1 + s T_a)}\right]$$

$$+ \frac{1}{R_3} \left[\frac{(R_1 T_i + R_i T_1) s + (R_1 + R_i)}{R_1 R_i}\right]$$

which on reduction to common denominator, rearrangement and simplification becomes:-

$$\beta = \frac{a_1 s^3 + b_1 s^2 + c_1 s + d}{R_1 R_2 R_3 R_i (1 + s T_a)} \qquad (34)$$

where

$$\begin{aligned} a_1 &= R_3 T_2 T_a (R_1 T_i + R_i T_1) \\ b_1 &= R_3 T_a (R_1 T_i + R_i T_1) + T_2 \{R_i R_3 (T_1 + T_a) + R_1 R_3 (T_i + T_a) + R_1 R_i T_a\} \\ &\quad + R_2 T_a (R_1 T_i + R_i T_1) \\ c_1 &= R_i R_3 (T_1 + T_a) + R_1 R_3 (T_i + T_a) + R_1 R_i T_a \\ &\quad + T_2 \{R_i R_3 + R_1 R_3 + R_1 R_i + K R_1 R_i\} \\ &\quad + R_2 \{R_1 T_i + R_i T_1 + (R_1 + R_i) T_a\} \\ d_1 &= R_i R_3 + R_1 R_3 + R_1 R_i + R_2 (R_1 + R_i) + K R_1 R_i \end{aligned} \qquad (35)$$

Substituting equations (32) and (34) in (25)

$$\frac{E_0}{E_i} = - R_i \frac{(A + B s)}{(C + D s)} \qquad (36)$$

where

$$\left.\begin{aligned} A &= (d - b s^2) \\ B &= (-c - a s^2) \\ C &= (d_1 + b_1 s^2) \\ D &= (c_1 + a_1 s^2) \end{aligned}\right\} \qquad (37)$$

Now substituting $s = j\omega$ in equation (36)

$$\left[\frac{E_0}{E_i}\right]_{s=j\omega} = - R_i \frac{(A + j\omega B)}{(C + j\omega D)} \qquad (38)$$

Since K is very large, R_1, R_2, and R_i very much greater than R_3, T_a greater than T_1, T_2 and T_i, and for values of ω in the region of practical interest ($f \leqslant$ 10 Kc/sec)

$$d \gg b\,\omega^2$$

and therefore from equations (37) and (33):-

$$A \cong K R_2 \qquad (39)$$

$$c \gg a\,\omega^2$$

and therefore from equations (37) and (38):-

$$B \cong T_1 K R_2 \qquad (40)$$

$$d_1 \gg b_1\,\omega^2$$

and therefore from equations (35) and (37):-

$$C \cong K R_1 R_i \qquad (41)$$

$$c_1 \gg a_1\,\omega^2$$

and therefore from equations (35) and (37):-

$$D = T_2 K_1 R_i + R_1 R_i T_a + R_2 (R_1 + R_i) T_a \qquad (42)$$

On substituting equation (39) through (42) in (38) and simplifying:-

$$\left.\frac{E_0}{E_i}\right|_{j\omega} \cong - \frac{R_2}{R_1} \frac{(1 + j\omega T_1)}{1 + j\omega \{T_2 + \frac{T_a}{K}(1 + \frac{R_2}{R_1} + \frac{R_2}{R_i})\}} \qquad (43)$$

and the phase distortion, since the angles are small, is therefore

$$\phi \cong \omega\,[T_1 - \{T_2 + \frac{T_a}{K}(1 + \frac{R_2}{R_1} + \frac{R_2}{R_i})\}] \qquad (44)$$