$$\Delta P = A_8 A_7' A_6' A_5' A_4' A_3' A_2' A_1' + A_8 A_7 A_6 A_5 A_4 A_3 A_2 A_1. \quad (28)$$

Fig. 4 is a block diagram of information flow. The control counter is shown as a serial counter although it may be replaced by a parallel counter. Also, the diode nets are shown as clocked, although in practice it may be the flip-flops which are clocked. Similar results would have been obtained if set and reset equations had been used instead of change equations.

Applications

This same method may be extended to generate other variables by changing slope counter equations. Also, for situations where a closer approximation is necessary, slopes with multiples of (1/256) or (1/512) etc., could be used. But it would be necessary to use a 3- or 4-bit adder instead of the simple 2-bit adder shown.

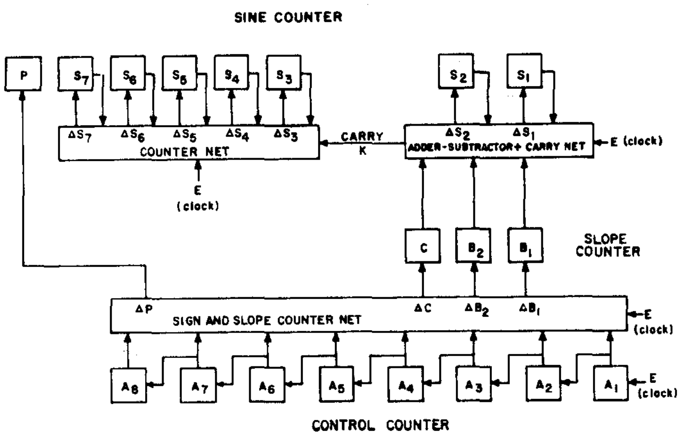

Fig. 4—Block diagram of information flow in the programmed counter for generating the sine function.

A Time-Division Multiplier*

M. LEJET LILAMAND†

Summary—**A time-division multiplier for analog computers is described. Its features, for a switching pulse frequency of 2,000 cycles per second, are as follows: an accuracy of one part in a thousand, a pass band of 2 cycles per second, an input impedance of one megohm, and a very low output impedance (the output impedance of a feedback amplifier).**

This multiplier has the following advantages when compared to two other types of analog multipliers: a) an accuracy limited solely by the stability of the components used and the fineness of the adjustments that can be made; b) a pass band greater than that of servomechanism multipliers; c) a much smaller amount of material than is necessary for diode multipliers with translators having parabolic characteristics and adjustments which can be made much more rapidly (although requiring a certain amount of practice); d) the possibility of changing the diodes without having to repeat all the adjustments.

These results have been obtained by the development of a precision electronic switch and by compensation of the stray capacities of the tubes.

Introduction

In analog computing machines, the primary form of nonlinear operation is multiplication. The Société d'Électronique d'Automatisme uses three types of multipliers: the servomechanism multiplier; the diode multiplier driving a translator having a parabolic characteristic; and finally, the modulation or time-division multiplier which is the subject of this article. This latter multiplier is of interest because it is more precise than the servomechanism multiplier. It is distributive and it requires much less material than the diode multiplier.

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The basic idea resides in the product of two modulations, one of the pulse-duration and the other of amplitude, of the form proposed in particular by Goldberg.

The multipliers based upon this principle[1] have, until the present time, been of insufficient accuracy or else they have been much too complex. At the Société d'Électronique d'Automatisme we have found a solution which is both economical and precise and which was made possible by the development of a precision electronic switch[2] and by the compensation of the stray capacities of the tubes.

We have also been able to obtain an accuracy of 0.1 per cent for an average switching pulse frequency of 1.5 kc.

Principle of Operation

The simplified schematic shown in Fig. 1 does not correspond exactly to the actual circuit, but it allows a clearer understanding of the principle of operation.

E_0 is a reference voltage ($E_0 = 100$ volts).

x and y are the two numbers, having values between -1 and $+1$, which are to be multiplied.

$E(x, y)$ and $E'(x, y)$ are the inputs (having values between 0 and E_0) to the switches W and W'.

Z and Z' are two other continuous inputs.

W and W' are two electronic switches controlled in synchronism by the flip-flop circuit B. (Actually, B

[1] See the bibliography.

[2] F. H. Raymond, "Improvements in electronic switching," Patent P.V. 658.091, November 13, 1953, and F. H. Raymond and M. B. Lejet, "Improvements applied to electronic switches," Patent P.V. 659.013, November 28, 1953.

consists of a sensing device—the flip-flop proper and a power stage.)

The operation is such that the output voltage U of the integrator A oscillates between two limits, U_1 and U_2, which are fixed in advance by the flip-flop and which are reached successively.

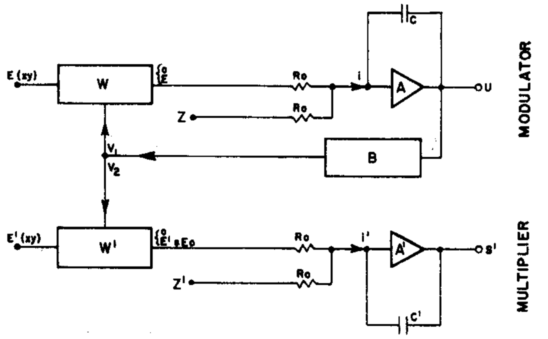

Fig. 1—Schematic diagram showing the operating principles of a time-division multiplier.

In order to clarify the operation, let us consider it as taking place in two phases:

Phase I: B delivers the control voltage $V_c = V_1$, W delivers voltage 0, W' delivers voltage 0.

Phase II: B delivers the control voltage $V_c = V_2$, W delivers voltage E, W' delivers voltage E'.

Let us consider the multiplier in operation. At the instant of time t_0+0, $U = U_2$ and B has just flipped. This is the beginning of the first phase (Fig. 2).

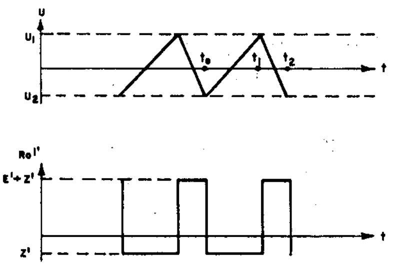

Fig. 2—Switching of the signal with time. U is the output voltage of the modulator, $R_0 i'$ is the input voltage to the multiplier.

Phase I:

$$U = U_2 - \frac{1}{R_0 C}\int_{t_0}^{t} Z dt \qquad t_0 \geqq t.$$

If we keep $Z<0$, U increases. Let t_1 be the instant at which $U = U_1$. We then have:

$$U_1 = U_2 - \frac{1}{R_0 C}\int_{t_0}^{t^1} Z dt.$$

Let us take the time $T_1 - t_0$ sufficiently small so that Z may be considered constant:

$$U_1 = U_2 - \frac{1}{R_0 C} Z T_1. \qquad (1)$$

At time t_1, a new switching action takes place and the system enters its second phase.

Phase II:

$$U = U_1 - \frac{1}{R_0 C}\int_{t_0}^{t} (E+Z) dt.$$

If we keep $(E+Z)>0$, U decreases until time t_2 at which $U = U_2$. We have then:

$$U_2 = U_1 - \frac{1}{R_0 C}\int_{t_1}^{t_2} (E+Z) dt$$

$$= U_1 - \frac{1}{R_0 C}(E+Z)T_2. \qquad (2)$$

B flips and the cycle starts over again.

Eqs. (1) and (2) may be written as follows:

$$\lambda = R_0 C(U_1 - U_2) = -ZT_1 = (E+Z)T_2,$$

from which:

$$-\frac{Z}{T_2} = \frac{E+Z}{T_1} = \frac{E}{T_1+T_2}$$

$$\frac{T_1}{T_1+T_2} = \frac{E+Z}{E} \qquad (3)$$

$$\frac{T_2}{T_1+T_2} = -\frac{Z}{E}, \qquad (4)$$

and the switching frequency is:

$$F = \frac{1}{T_1+T_2} = \frac{1}{\lambda\left(\frac{1}{-Z} + \frac{1}{E+Z}\right)} = \frac{-Z(E+Z)}{\lambda E}. \qquad (5)$$

The average value, during one cycle, of the current i' which enters the integrator A' is:

$$i_m' = \frac{1}{R_0}\left(\frac{E'T_2}{T_1+T_2} + Z'\right) = \frac{1}{R_0}\left(-Z\frac{E'}{E} + Z'\right).$$

The input voltage to the multiplier, referred to a resistance R_0, is:

$$E_m' = R_0 i_m' = -Z\frac{E'}{E} + Z'.$$

It is desirable that this voltage be proportional to xy, in which case it is necessary that:

E be independent of x and of y, and of the form: $E = cE_0$;

Z be a linear function of x, of the form: $Z = (ax - b)E_0$;

E' be a linear function of y, of the form: $E' = (a'y + b')E_0$.

(The signs of a and a' are unimportant. We may, for example, assume them to be positive.)

$$E_m' = \frac{(b - ax)(a'y + b)}{c}E_0 + Z'$$

$$= -\frac{aa'}{c}xy - \frac{ab'x - ba'y - bb'}{c}E_0 + Z'.$$

It is further necessary that:

$$Z' = \frac{ab'x - ba'y - bb'}{c}.$$

Let us find the optimum values for a, a', b, b', and c. It would be desirable to have $a'a/c$ as large as possible (amplifier A' operated with small gain); in other words, the two quantities a/c and a' must both be large. In addition, the coefficients a, a', etc., must obey the conditions previously encountered, namely:

switch inputs:

$$0 < E < E_0 \text{ or } 0 < c < 1$$

$$0 < E' < E_0 \text{ or } 0 < (b' - a') < (b' + a') < 1.$$

This latter condition may be written:

$$\begin{cases} a' < b' \\ (b' + a') < 1 \end{cases}$$

where a' is as large as possible. Thus, it is necessary to take $a' = b' = \frac{1}{2}$. On the other hand we have:

$$0 < \frac{T_1}{T_1 < T_2} < 1,$$

but since it could be inconvenient to have $T_1 = 0$ or $T_2 = 0$, we restrict ourselves as follows:

$$0.1 < \frac{T_1}{T_1 + T_2} < 0.9$$

$$0.1 < \frac{-Z}{E} < 0.9$$

$$0.1 < \frac{b - ax}{c} < 0.9$$

$$0.1 < \frac{b-a}{c} < \frac{b+a}{c} < 0.9$$

$$\left.\begin{aligned} \frac{a}{c} + \frac{b}{c} &< 0.9 \\ \frac{a}{c} - \frac{b}{c} &< -0.1 \end{aligned}\right\} \quad (6)$$

which yields:

$$a/c < 0.4.$$

We may take, for example, $c = 1$ and $a = \frac{1}{3}$ in order to have simple numbers. The inequalities (6) become:

$$\begin{cases} b/c < 0.9 - 0.33 \\ b/c > 0.1 + 0.33 \end{cases}$$

$$\text{or } 0.43 < b/c < 0.57.$$

Thus we can take the value $b/c = \frac{1}{2}$.

Summarizing, if we start with the following values:

$$a = \tfrac{1}{3} \qquad b = \tfrac{1}{2}$$

$$a' = \tfrac{1}{2} \qquad b' = \tfrac{1}{2}$$

and if input voltages (referred to a resistance R_0) are:

$$E = E_0, \qquad Z = \left(\frac{x}{3} - \frac{1}{2}\right) E_0 \qquad (7)$$

$$E' = \frac{1 + y}{2} E_0 \quad Z' = \left(\frac{x}{6} - \frac{y}{4} - \frac{1}{4}\right) E_0, \qquad (8)$$

then the average value of the input to the multiplier will be:

$$E_m = -\frac{xy}{6} E_0.$$

and it will suffice to set the amplifier A' for a gain of 6 in order to have it deliver an output of:

$$xyE_0.$$

Equipment

The multiplier consists of two parts: the modulator (upper part of Fig. 1), where the pulse width or duration is modulated, and the multiplier proper (lower part of Fig. 1) where the amplitude modulation takes place. The heart of both the modulator and the multiplier is the electronic switch.

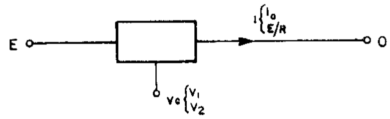

Fig. 3—Block diagram of a switch. E is the input, i is the output, V_c is the control voltage.

Electronic Switch Using Diodes

The operation of the electronic switch is shown in block form in Fig. 3.

At the input to the switch, we have a positive voltage E which varies from E_{min} to E_{max} (5 to 100 volts). At the output, we have zero voltage and a current i such that:

$i = i_0$ independent of E, when $V_c = V_1$;

$i = E/R$ proportional to E, when $V_c = V_2$.

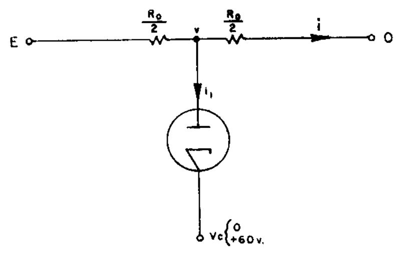

Fig. 4—Schematic diagram of a switch using a single diode.

The simplest system which comes to mind is that shown in Fig. 4.

Here $i = 0$ when $V_c = 0$ (diode is conducting) and $i = E/R_0$ when $V_c = +60v$ (diode is blocking).

In reality, when the diode conducts, the plate voltage v of the diode is not actually zero. There is a residual voltage drop which is dependent upon E.

The current in the diode is:

$$i_1 = \frac{E - v}{\frac{R_0}{2}} - \frac{v}{\frac{R_0}{2}},$$

from which:

$$v = \frac{E}{2} - \frac{R_0}{4} i_1. \tag{9}$$

On the diode characteristic, shown in Fig. 5, we can see the variation Δv of v as E varies from $E_{\min}$ to $E_{\max}$. If the characteristic were linear in this region, one could compensate for this variation, but it is not.

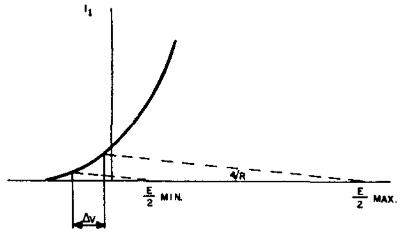

Fig. 5—Operating point of a switch using a single diode.

It can be shown that Δv reaches 250 mv under the following conditions:

Diode EB41: 6 volt filament,
$R_0 = 1$ megohm,
$5v < E < 100v$.

The switch which was finally decided upon, after what was believed to be a complete investigation of the subject, is shown in Fig. 6. It consists of two switches identical to the one just described. The second switch (consisting of R_2, R_3, and D_2 in Fig. 6) improves the characteristics of the first switch (consisting of R_1, R_2, and D_1 in Fig. 6) when both diodes are conducting.

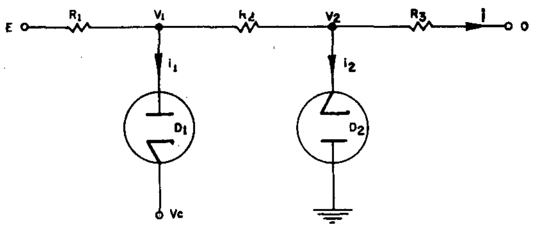

Fig. 6—Diagram of a switch using two diodes.

The first diode is biased negatively ($V_c = V_1 = -11v$) causing (when both diodes are conducting) a current i_2 to flow in the second diode which is independent of E. The operation may be seen on the graph of Fig. 7.

$$i_1 = \frac{E - v_1}{R_1} - \frac{v_1 - v_2}{R_2},$$

v_1 being the plate voltage of the first diode and
v_2 being the cathode voltage of the second diode.

One could begin by neglecting $v_2 (v_2 \approx 0.1$ volt) in order to compute $v_1 (v_1 \approx -11$ volts), from which we get:

$$v_1 = \frac{E}{1 + \frac{R_1}{R_2}} - i_1 \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \tag{10}$$

which gives us the points A and B (Fig. 7). Moreover:

$$i_2 = \frac{v_1 - v_2}{R_2} - \frac{v_2}{R_3} \tag{11}$$

$$v_2 = \frac{v_1}{1 + \frac{R_2}{R_3}} - i_2 \frac{1}{\frac{1}{R_3} + \frac{1}{R_3}} \tag{12}$$

which gives us the points C and D (Fig. 7).

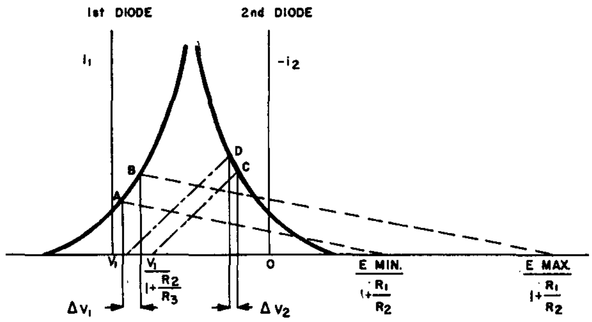

Fig. 7—Operating point of a switch using two diodes.

The variation Δv_2 of voltage v_2 will thus be less than 1 mv under the following conditions:

Diode EB41: 6 volt filament,

$5V < E < 100V \qquad v_1 = -11v,$

$R_1 = 0.5$ megohm, $R_2 = 0.1$ megohm, $R_3 = 0.4$ megohm.

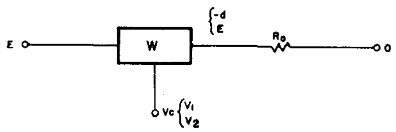

Fig. 8—Equivalent diagram of the switch.

Summarizing, the electronic switch (Fig. 8) will act as a resistance $R_0 = R_1 + R_2 + R_3$ to which a voltage is applied equal to:

$-d$ when $V_c = V_1$ (diodes conducting),
E when $V_c = V_2$ (diodes blocking),

where, by definition:

$$-d \times \frac{R_3}{R_1 + R_2 + R_3} = v_2. \tag{13}$$

The minus sign is chosen because in general v_2 is negative; in this case, since the current i_2 flowing through the

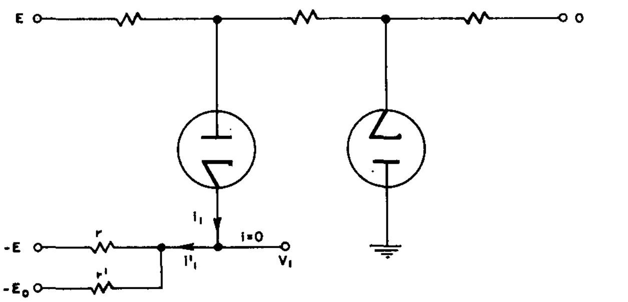

Fig. 9—Compensation of the current i_1 flowing through the diode, by a current i_1' having an opposite sense. V_1 is a source of voltage whose internal impedance is not zero.

second diode is independent of E, it is possible to cancel the residual voltage v_2 by connecting in series with the diode a variable resistance of the order of magnitude of the diode resistance without detracting from the qualities of the switch.

All this assumes that the voltage v_1 remains constant, whatever may be the value of the current i_1 flowing through the diode. In fact, the voltage source V_1 (cathodyne output of a power stage) has an internal resistance which is not negligible (5,000 ohms) and so, by using the arrangement shown in Fig. 9, a current i_1', equal to, and of opposite sign from, i_1, is caused to flow through the source simultaneously with current i_1, so that the total current will be approximately zero.

From (5):

$$i_1 \approx \frac{E}{R_1} - V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \tag{14}$$

$$i_1' = \frac{V_1 + E}{r} + \frac{V_1 + E_0}{r'}. \tag{15}$$

It is necessary that $i_1 \equiv i_1'$ for every value of E. In other words:

$$\begin{cases} R_1 = r & (16) \\ -V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_1}{r} + \frac{V_1 + E_0}{r'} & (17) \end{cases}$$

$$r = R_1 \tag{18}$$

$$r' = \frac{E_0 + V_1}{-V_1\left(\frac{2}{R_1} + \frac{1}{R_2}\right)} \quad (V_1 \text{ is negative}). \tag{19}$$

Modulator[3]

The modulator circuit is shown in Fig. 10.

During the period T_1, the diodes conduct and the input voltage, referred to a resistance R_0, is (Fig. 11):

$$\frac{xE_0}{3} \quad \text{for the first branch}$$

$$\frac{-(E_0 - 2d)}{2} \quad \text{for the second branch}$$

[3] F. H. Raymond, "Driver for the electronic analogue computer, particularly for the multiplier," Patent P.V. 659.014, November 28, 1953.

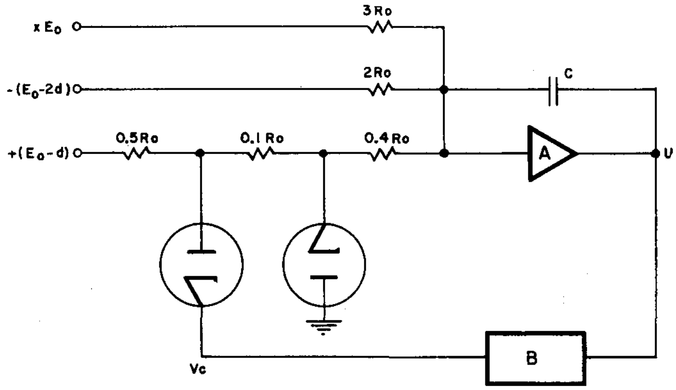

Fig. 10—Diagram of a modulator. A is an infinite gain amplifier shunted by C. B is a flip-flop with two stable stages.

$$-d \quad \text{for the third branch}$$

$$= \left(-\frac{1}{2} + \frac{x}{3}\right)E_0 \text{ for the total input.} \tag{20}$$

During period T_2, the diodes are blocking and the input voltage, again referred to a resistance R_0, is (Fig. 11):

$$\frac{xE_0}{3} \quad \text{for the first branch}$$

$$\frac{-(E_0 - 2d)}{2} \quad \text{for the second branch}$$

$$+E_0 - d \quad \text{for the third branch}$$

$$= \left(+\frac{1}{2} + \frac{x}{3}\right)E_0 \text{ for the total input.} \tag{21}$$

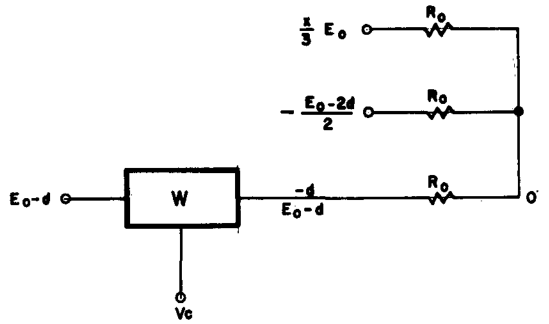

Fig. 11—Equivalent diagram of a modulator. The input resistances are taken as equal to R_0.

In practice, in order to obtain the voltages (E_0-d) and $-(E_0-2d)$, (where d is a positive voltage), the circuit shown in Fig. 12(a) is replaced by its equivalent circuit, Fig. 12(*b*), after properly adjusting the potentiometers $P+$ and $P-$ so as to produce the same current.

Referring now to the principle of operation mentioned earlier, while letting $E = E_0$ and $Z = (x/3)E_0 - \frac{1}{2}(E_0 - 2d) - d = (x/3 - \frac{1}{2})E_0$, (3) and (4) become:

$$\frac{T_1}{T_1 + T_2} = \frac{1}{2} + \frac{x}{3} \tag{3'}$$

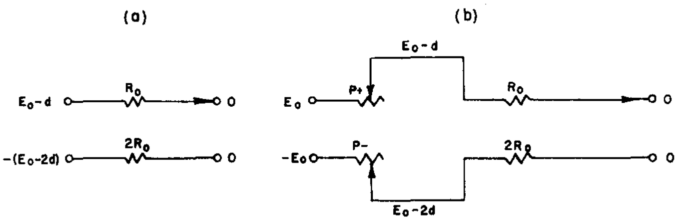

Fig. 12—Compensation of the voltage d

$$\frac{T_2}{T_1 + T_2} = \frac{1}{2} - \frac{x}{3}. \qquad (4')$$

The pulse repetition frequency F can be calculated from (5). Thus, we find that:

$$F = \frac{1}{T_1 + T_2} = \frac{E_0}{(U_1 - U_2)R_0C}\left(\frac{1}{4} - \frac{x^2}{9}\right). \qquad (22)$$

This frequency may be varied by causing C to vary.

Multiplier

The multiplier circuit is shown in Fig. 13.

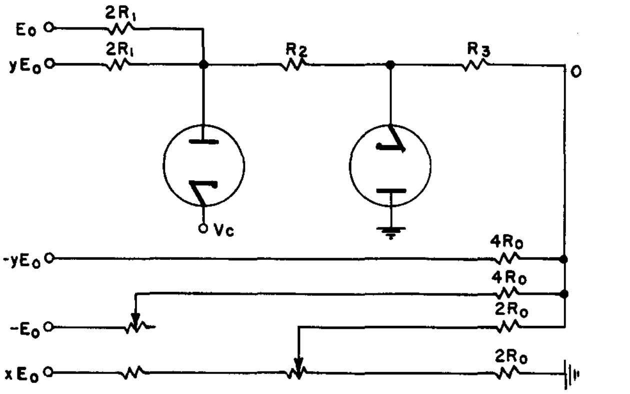

Fig. 13—Diagram of the multiplier. 0 is the grid (maintained at zero potential) of a negative feedback amplifier with an infinite gain.

Let us calculate the average input voltage E_m to the first branch, referred to a resistance R_0 (Fig. 14(a) and (b)):

Period T_1, diodes conducting, input $= -d'$

Period T_2, diodes blocking, input $= \frac{1}{2}(1 + y)E_0$.

$$E_m = \frac{-d'T_1 + \dfrac{1+y}{2}E_0T_2}{T_1 + T_2}. \qquad (23)$$

By using (3′) and (4′), it is found that

$$E_m = -\frac{xy}{6}E_0 + \frac{y}{4}E_0 - x\left(\frac{1}{6} + \frac{d'}{3E_0}\right)E_0 + \left(\frac{1}{4} - \frac{d'}{E_0}\right)E_0. \qquad (24)$$

It will suffice to add three other unswitched inputs in order to compensate for all the terms which are different from $-(xy/6)E_0$ (as with the modulator, potentiometers are used to take the voltage containing d' into account). If the approximation $d'=0$ is made, the basic circuit of Fig. 1 is arrived at again. Note that the variation of y must be restricted within the following limits: $-0.9 < y < +1.0$. In effect, the input to the switched branch, $E = \frac{1}{2}(1+y)E_0$, cancels for $y = -1$. The first diode is thus effectively blocked for $V_c = V_2(+70v)$ but the second diode loads up (zero potential difference between plate and cathode).

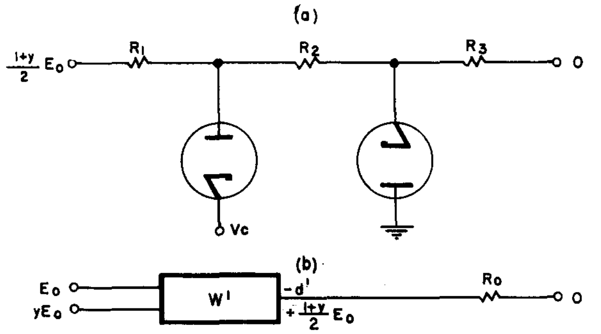

Fig. 14—(a) Switches branch of the multiplier; (b) Equivalent diagram of this branch, the input resistance taken as R_0.

With further reference to Fig. 1, according to whether the feedback circuit comprises a capacitor C' [Fig. 15(a)] or a resistor R [Fig. 15(b)] in parallel with a capacitor which is sufficiently large to act as a filter, voltages at the output S of the multiplier will be

$$S = \frac{1}{R_0C'}\int\frac{xy}{6}E_0dt \text{ or } S = \frac{R}{R_0}\frac{xy}{6}E_0, \text{ respectively.}$$

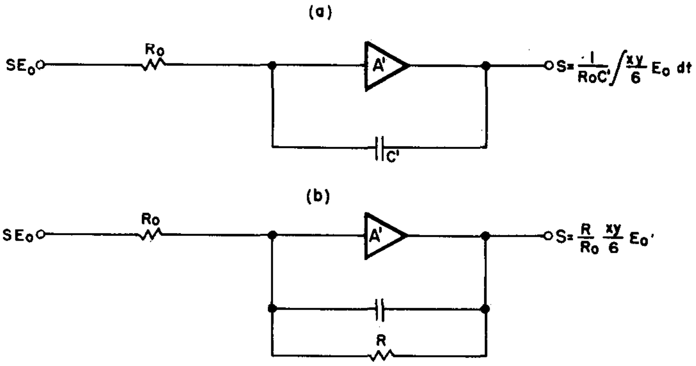

Fig. 15—Voltage at the output S of the multiplier. (a) Amplifier feedback via a capacitor; (b) amplifier feedback via a filter.

Compensation of Stray Capacities

The stray capacities are of the order of 5 pF. They produce important errors (several thousandths). They appear only because of the use of nonlinear elements. Fig. 16 shows the stray capacities C_1, C_2, C_3 of the modulator and C_1', C_2', C_3' of the multiplier.

Calculation of Time Constants: When the diodes are blocking, the capacities C_1, C_2, C_3 charge up and the current i does not reach its normal value instantaneously:

$$i = E/(R_1 + R_2 + R_3).$$

Thus the average value of i during the period T_2 is:

$$i = \frac{E}{R_1 + R_2 + R_3} - \frac{\Delta q}{T_2},$$

where Δq is the amount of electricity which serves to charge the stray capacities (to be calculated later).

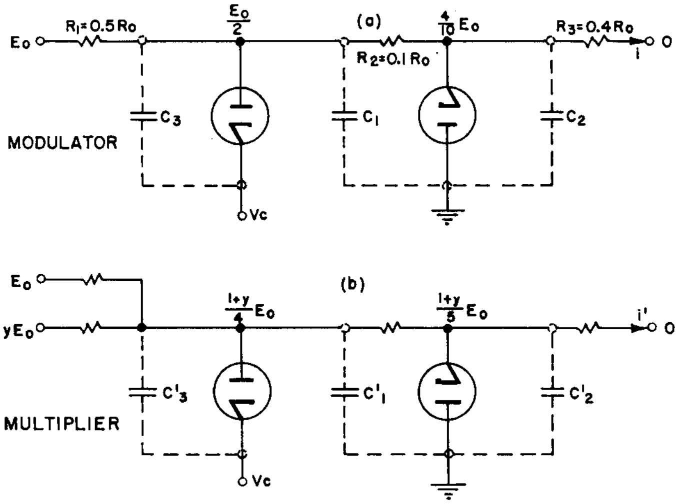

Fig. 16—Location of stray capacities (a) for the modulator and (b) for the multiplier.

When the diodes conduct, the capacities discharge across the diodes (low impedance) and the current i has for its normal value:

$$i = -d/(R_1 + R_2 + R_3).$$

We shall reexamine (1) and (2) which describe the operation of the modulator, but now we shall take Δq into account. For simplicity, we shall assume that the residual voltage d has been reduced to zero. We have:

$$\lambda = \left(\frac{1}{2} - \frac{x}{3}\right) T_1 = \left(\frac{1}{2} + \frac{x}{3}\right) T_2 - \Delta q \frac{R_0}{E_0},$$

from which:

$$\frac{\frac{1}{2} - \frac{x}{3}}{T_2} = \frac{\frac{1}{2} + \frac{x}{3} - \frac{\Delta q}{T_2}\frac{R_0}{E_0}}{T_1} = \frac{1 - \frac{\Delta q}{T_2}\frac{R_0}{E_0}}{T_1 + T_2}$$

$$\frac{T_2}{T_2 + T_1} = \frac{\frac{1}{2} - \frac{x}{3}}{1 - \frac{\Delta q}{T_2}\frac{R_0}{E_0}}$$

$$\approx \left(\frac{1}{2} - \frac{x}{3}\right)\left(1 + \Delta q \frac{R_0}{T_2 E_0}\right). \tag{25}$$

The term

$$\frac{\Delta q}{T_2}\frac{R_0}{E_0}$$

is a correction factor which is small compared to unity. In this factor, T_2 may be replaced by the value it has when there are no stray capacities:

$$\left(\frac{1}{2} - \frac{x}{3}\right)(T_1 + T_2) \quad \text{(refer to Fig 4).}$$

Whence:

$$\frac{T_2}{T_1 + T_2} = \left(\frac{1}{2} - \frac{x}{3}\right)\left[1 + \frac{\Delta q R_0}{E_0\left(\frac{1}{2} - \frac{x}{3}\right)(T_1 + T_2)}\right]$$

$$= \left(\frac{1}{2} - \frac{x}{3}\right) + \Delta q \frac{R_0}{E_0} F,$$

$$\text{where } F = \frac{1}{T_1 + T_2}. \tag{26}$$

Similarly, for (23) which describes the operation of the multiplier, we find (keeping $d' = 0$):

$$E_m = \frac{E_0\left(\frac{1+y}{2}\right) T_2 - \Delta g' R_0}{T_1 + T_2}$$

$$= E_0\left(\frac{1+v}{2}\right)\left(\frac{1}{2} - \frac{x}{3}\right) + \left(\frac{1+y}{2}\right)\Delta q R_0 F - \Delta q' R_0 F. \tag{27}$$

The error introduced by the capacities is thus:

$$\Delta E_m = R_0 F\left[\frac{1+y}{2}\Delta q - \Delta q'\right]. \tag{28}$$

Now we shall calculate Δq and $\Delta q'$, calling E the input to the switch ($E = E_0$ for the modulator and $E = \frac{1}{2}(1+y) \cdot E_0$ for the multiplier). The variation of charge for the capacity C_1 is:

$$\Delta Q_1 = C_1\left[E \frac{R_2 + R_3}{R_1 + R_2 + R_3} - V_1\right].$$

The quantity of electricity Δq_1, which will not pass through resistor R_3 because of the charge on C_1 is:

$$\Delta q_1 = \Delta Q_1 \frac{R_2}{R_2 + R_3} = C_1\left[E \frac{R_1}{R_0} - V_1 \frac{R_1}{R_2 + R_3}\right]$$

where $R_0 = R_1 + R_2 + R_3$.

Similarly for capacities C_2 and C_3:

$$\begin{cases} \Delta Q_2 = C_2 E \frac{R_3}{R_0} \\ \Delta q_2 = \Delta Q_z \frac{R_1 + R_2}{R_3} = C_2 E \frac{R_1 + R_2}{R_0} \end{cases}$$

$$\Delta Q_3 = C_3\left[E \frac{R_2 + R_3}{R_0} - V_2\right]$$

$$\Delta q_3 = \Delta Q_3 \frac{R_1}{R_2 + R_3} = C_3\left[E \frac{R_1}{R_0} - V_2 \frac{R_1}{R_2 + R_3}\right].$$

Whence:

$$\begin{aligned} R_0 \Delta q &= R_0(\Delta q_1 + \Delta q_2 + \Delta q_3) \\ &= E[R_1 C_1 + (R_1 + R_2) C_2 + R_1 C_3] \\ &\quad - \frac{R_1 R_0}{R_2 + R_3}(V_1 C_1 + V_2 C_3). \end{aligned} \tag{29}$$

This is of the general form:

$$R_0 \Delta q = aE - bE_0$$

where

$$a = R_1C_1 + (R_1 + R_2)C_2 + R_1C_3, \tag{30}$$

$$b = \frac{R_1R_0}{R_2 + R_3}\left[\frac{V_1}{E_0}C_1 + \frac{V_2}{E_0}C_3\right]. \tag{31}$$

a and b are homogeneous with the time constants.

In the case of the modulator $E = E_0$, so that

$$q = (a - b)E_0.$$

In the case of the multiplier: $E = \frac{1}{2}(1+y)E_0$

$$q' = (\tfrac{1}{2}a'(1 + y) - b')E_0.$$

The error for the multiplier is:

$$\Delta E_m = F\left[\frac{1+y}{2}(a - b)E_0 - \left(a'\frac{1+y}{2} - b'\right)E_0\right]$$

$$= E_0F\left[\frac{1+y}{2}(a - a' - b) + b'\right]. \tag{32}$$

The frequency F being proportional to $(\frac{1}{4} - x^2/9)$ from (22), the capacities introduce errors in the terms containing 1, x^2, y, x^2y.

Order of Magnitude of ΔE_m*:* Let us assume that the switch for the modulator is identical with the switch for the multiplier ($a = a'$, $b = b'$). An error exists of the order of magnitude of bE_0F.

Thus, if $F = 2{,}000$ cps (a large value of F is taken in order to increase the pass band of the multiplier):

$$R_1 = 0.5 \text{ meg } \Omega$$
$$R_2 = 0.1 \text{ megohm}$$
$$R_3 = 0.4 \text{ megohm}$$
$$C_1 = C_2 = C_3 = 5pF$$
$$V_1 = -11V$$
$$V_2 = +70V$$

and we have

$$\frac{\Delta E_m}{E_0} = bF = 6 \times 10^{-3}.$$

Let us examine the expression for the error ΔE_m, (32). In order to cancel ΔE_m, it is necessary, in the first place, to increase the modulator time constant by Δa so that:

$$a + \Delta a - a' - b = 0.$$

This is accomplished by inserting a variable capacitor C_0 in parallel with C_2. Thus we have:

$$\Delta a = (R_1 + R_2)C_0.$$

In the second place, it is necessary to compensate for the term E_0Fb'. This is simple because the signal fed into the cathode of the diode consists of rectangular pulses having a constant amplitude of $(V_2 - V_1)$ and a frequency F. It is sufficient to connect into the multiplier circuit (Fig. 17) a variable capacitor C_4' and a rectifier in order to obtain the proper sign.

The charge variation of the capacitor C_4' is:

$$\Delta Q_4 = C_4'(V_2 - V_1).$$

The charging current of the capacitor flows through the rectifier (which we will assume to be perfect) while the discharge current flows through resistor r. The average current $\Delta i_{4m}'$ which flows through r will be:

$$\Delta i_{4m}' = -FC_4'(V_2 - V_1). \tag{35}$$

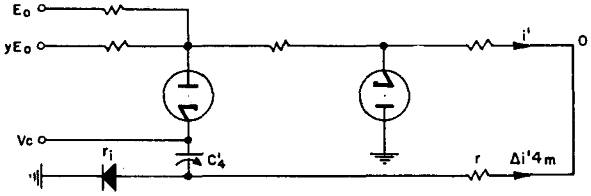

Fig. 17—Corrective capacitor C_4' connected in the multiplier.

This current adds to the multiplier current i'. It is equivalent to an input voltage of $\Delta i_{4m}'R_0$.

Thus it is necessary that:

$$b'E_0F - FC_4'(V_2 - V_1)R_0 = 0 \text{ and}$$

$$C_4' = \frac{b'E_0}{R_0(V_2 - V_1)}.$$

Method of Adjustment. It should be noted that the above mentioned capacities introduce errors which are proportional to the switching frequency F. In order to reduce the errors, it would be sufficient *a priori* to reduce the frequency, but in doing so, the pass band is simultaneously reduced since it is the average values of x and y during the periods T_1 and T_2 which are important.

For a frequency of 2,000 cps, the pass band is about 2 cps. In order to increase it, it is sufficient to decrease the value of the modulator feedback capacitor C.

Since a modulator must be capable of controlling several multipliers, the capacitor C_0 is given *a priori* a larger value, so that, whatever may be the multiplier:

$$a + \Delta a - a' - b > 0, \quad \text{where} \quad a = (R_1 + R_2)C_0.$$

If we replace F by its value in (32), then the input error is given by the following:

$$\Delta E_m = E_0\left(\frac{1}{4} - \frac{x^2}{9}\right)\frac{E_0}{(U_1 - U_2)R_0C} \cdot \left[\frac{1+y}{2}(a + \Delta a - a' - b) + b'\right].$$

If the amplifier is adjusted for a gain of -6 in order to have $1 \times 1 = 1$, S may be written as follows:

$$S = xyE_0 - \frac{6E_0^2}{(U_1 + U_2)R_0C}\left(\frac{1}{4} - \frac{x^2}{g}\right) \cdot \left[\left(\frac{a + \Delta a - a' - b}{2} + b'\right) + \frac{a + \Delta a - a' - b}{2}y\right]$$

$$S = xyE_0 + A\left(\frac{1}{4} - \frac{x^2}{g}\right) + B\left(\frac{1}{4} - \frac{x^2}{g}\right)y$$

where A and B are functions of time constants a, a', etc.

If we make the following products:

$$x = 0 \qquad y = +1$$
$$x = 0 \qquad y = -1$$

we shall find:

$$S_1 = \frac{A}{4} + \frac{B}{4}$$
$$S_2 = \frac{A}{4} - \frac{B}{4}.$$

Therefore: $B = 2(S_1 - S_2)$.

The term B can be compensated by adjusting the variable capacity C_0' which is connected in parallel with C_2' in the multiplier. In other words, the time constant a' is modified in such a way as to have:

$$S_1 - \frac{B}{4} = S_1 - \frac{S_1 - S_2}{2} = \frac{S_1 - S_2}{2}.$$

When B is compensated, we have:

$$S = xyE_0 + A\left(\frac{1}{4} - \frac{x^2}{g}\right);$$

for $x=0$, $y=0$ we shall find: $S_3 = A/4$.

This term can be cancelled by adjusting C_4'; in other words b', which does not change B.

It should be noted that it is necessary to make these adjustments only once, at the time when these equipments are placed in operation. All further drifting of the multiplier (caused by variations of resistance with temperature or aging of the diodes) does not involve any readjustment of the capacitors.

Example of the Use of a Modulation Multiplier

An essential property of the multiplier is its *distributivity*. This property is particularly useful when vector products are to be computed. In aerodynamics, for example, it is desired to obtain the integral of the components of vector products of the form:

$\overline{W} \times \overline{V}$ (in the equation for the magnitude of motion)
$\overline{W} \times \overline{H}$ (in the equation for kinetic moment).

If the components p, q, r of the vector $\overline{W}$ feed three modulators, the control voltage coming out of each modulator can control as many multipliers as may be necessary.

We see in Fig. 18 how to perform the multiplication $\overline{W} \times \overline{V}$, that is, how to find the components:

$$\begin{cases} qw - rv, \\ ru - pw, \\ pv - qu. \end{cases}$$

With the same modulators, but with three other multipliers, we can similarly find $\overline{W} \times \overline{H}$. To obtain the 12 products, we need the following equipment:

3 amplifiers and 3 modulators,
3 amplifiers and 6 multipliers (for $\overline{W} \times \overline{H}$),
3 amplifiers and 6 multipliers (for $\overline{W} \times \overline{V}$),

or a total of 9 amplifiers, 3 modulators, and 12 multipliers—whence the great economy in the number of necessary amplifiers and modulators.

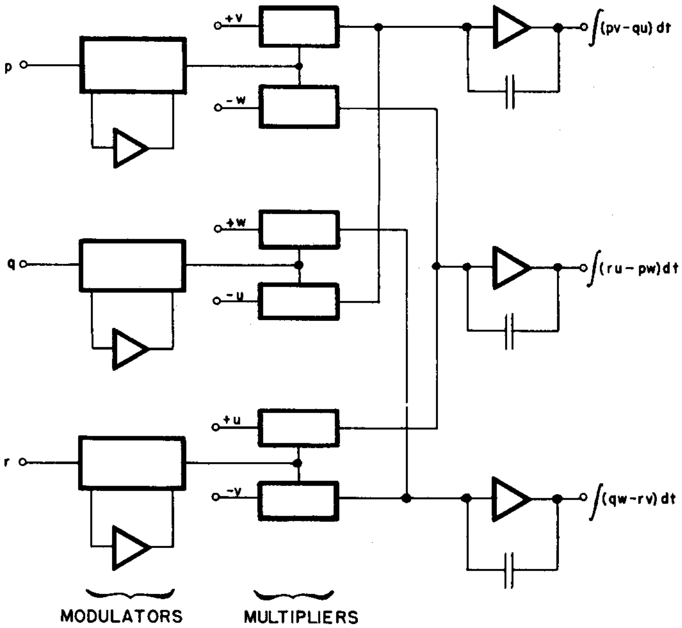

Fig. 18—Diagram illustrating the calculation of vector products.

Results and Conclusion

If the amplifier A' [Fig. 15(b)] is adjusted for a gain of 6 in order to have the following relation: $S = xyE_0$, the accuracy is $\Delta S/E_0 = 1/1{,}000 = 0.1$ per cent.

This accuracy is limited by the precision of the resistors and by the stability of the tubes (the variation of the diode characteristics because of aging or mechanical shock). The multiplier developed by Goldberg in the United States achieves independence of the tube characteristics by using current control rather than voltage control, but it has the disadvantage of requiring 6 amplifiers.

That is why, except for a slight decrease in accuracy, we undertook at Société d'Électronique d'Automatisme to find a simpler solution requiring only two amplifiers.

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